Error-controlled adaptive damage-to-fracture approach for modeling a complex failure in quasi-brittle materials

Challenges and first results

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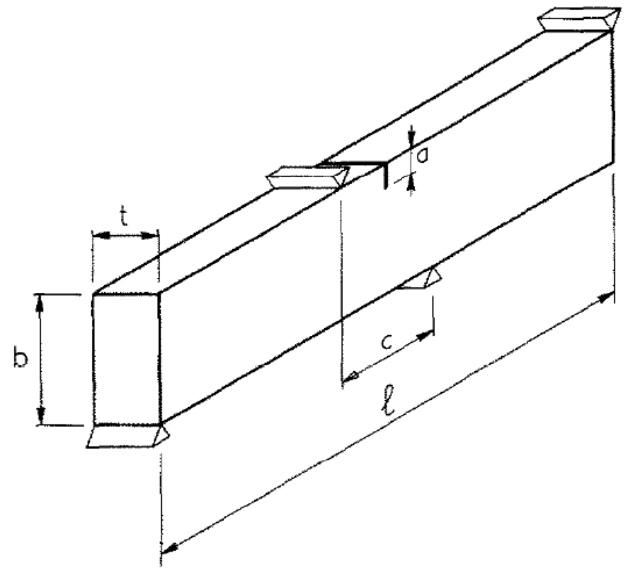
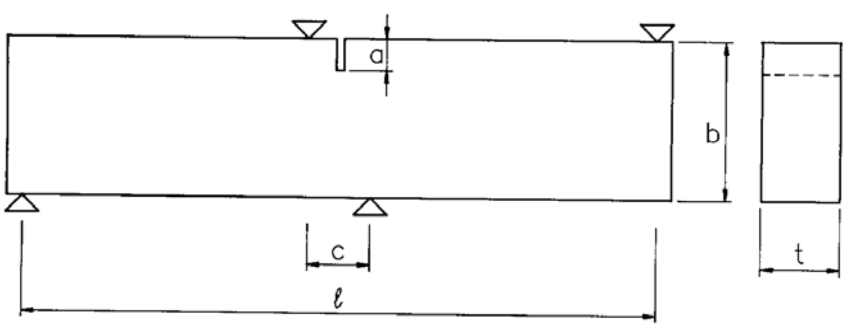
The financial support of DFG (project WR 19/39-1 & 2, Wriggers, Stein) is gratefully acknowledged

Failure of SEN concrete beam under asymmetric 4-point bending

Schlangen (1993) – “small” specimen, normal weight concrete





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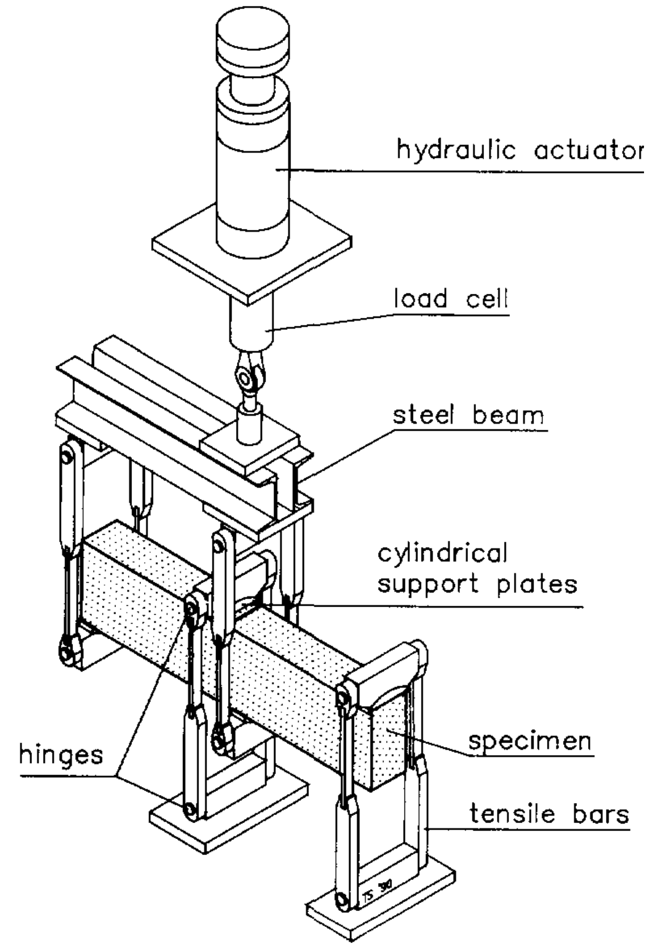
b [mm] = 100
c [mm] = 40
l [mm] = 400
t [mm] = 100



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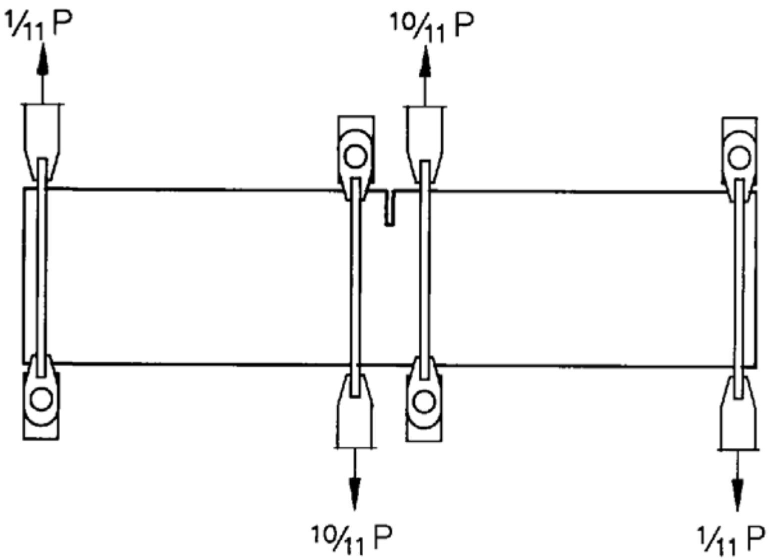

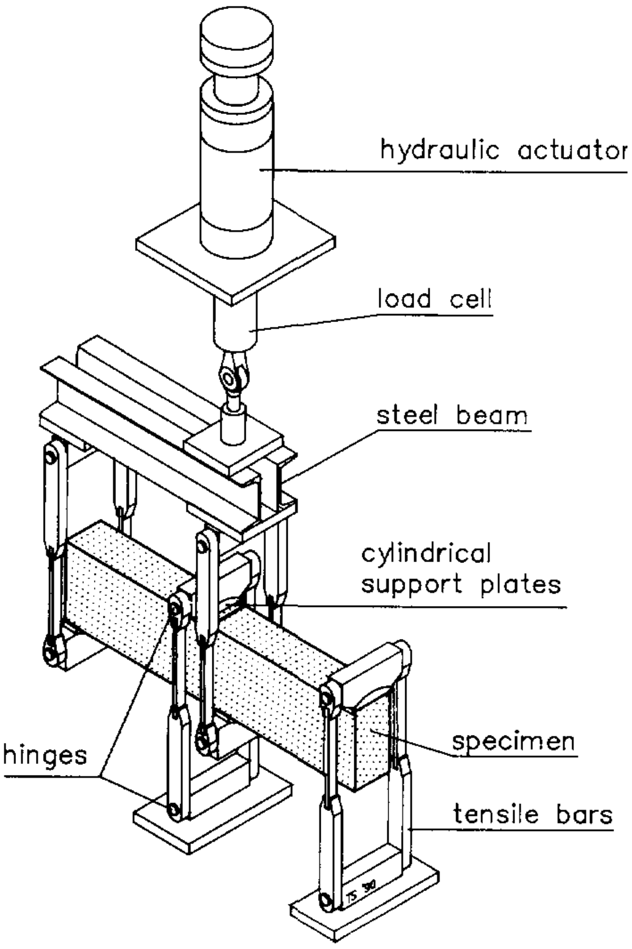

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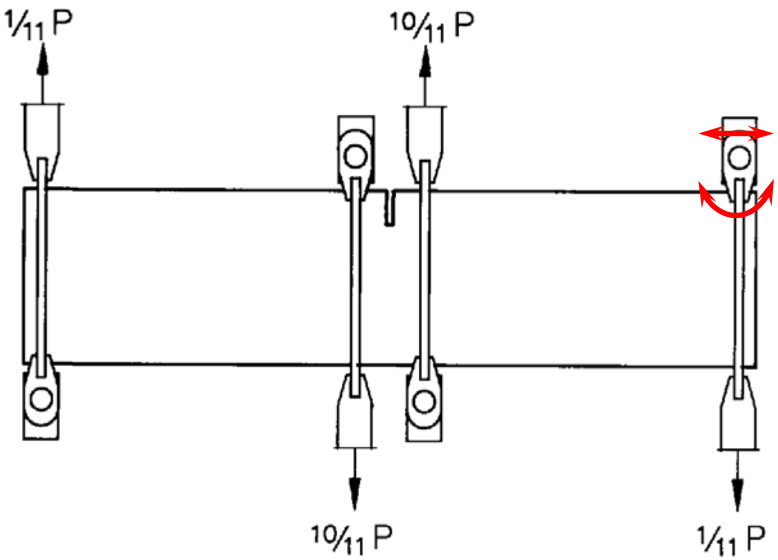
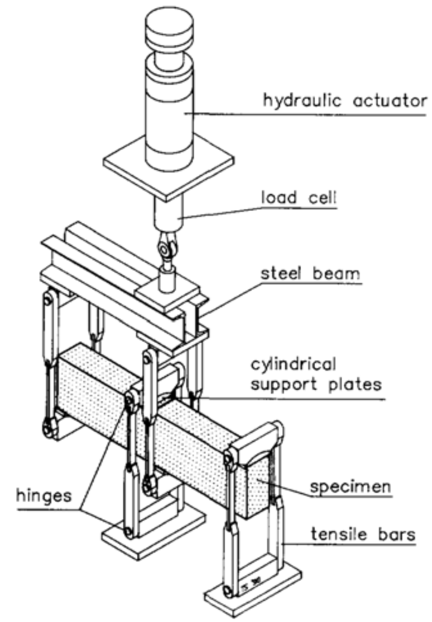
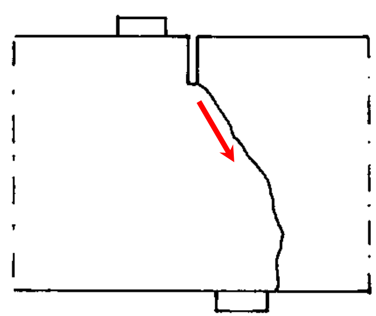




rotating supports



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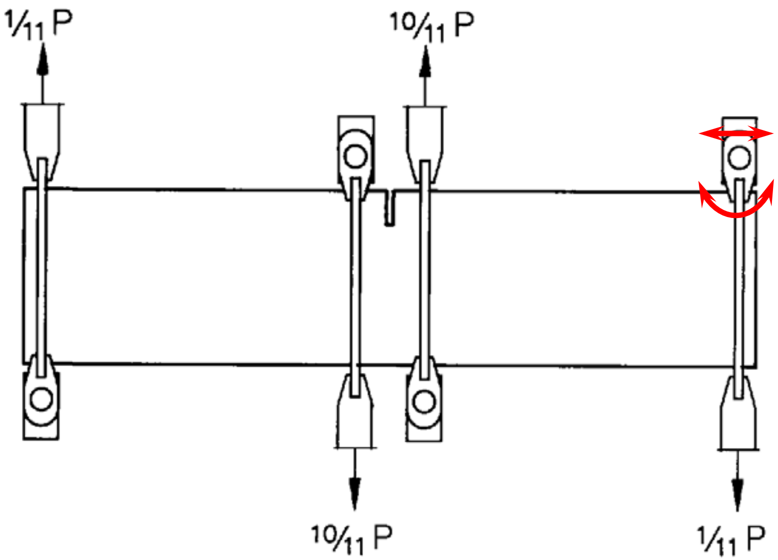
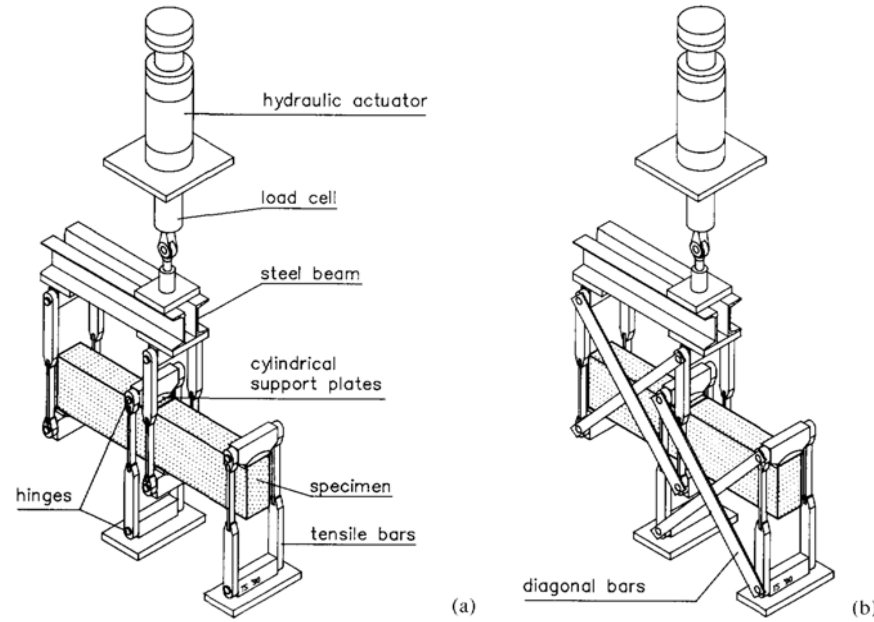
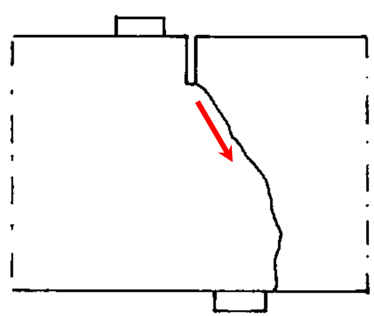
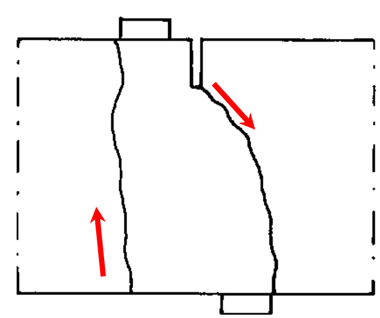




rotating supports

fixed supports





Failure of SEN concrete beam under asymmetric 4-point bending

Peerlings (1996), Geers *et al.* (2000) – gradient-enhanced damage model for concrete failure

$$D(\bar{\varepsilon}) := \begin{cases} 1 - \frac{\kappa_0}{\bar{\varepsilon}}\left(1 - \alpha + \alpha e^{-\beta(\bar{\varepsilon}-\kappa_0)}\right), & \bar{\varepsilon} > \kappa_0 \\ 0, & \bar{\varepsilon} \leq \kappa_0 \end{cases}$$

$$\begin{cases} \boldsymbol{\sigma} = (1 - D(\bar{\varepsilon}))\,\mathbb{C} : \boldsymbol{\varepsilon} \quad \text{in } \Omega \\ \begin{cases} -c\nabla^2\bar{\varepsilon} + \bar{\varepsilon} = \tilde{\varepsilon} \quad \text{in } \Omega \\ \nabla\bar{\varepsilon} \cdot \boldsymbol{n} = 0 \quad \text{on } \partial\Omega \end{cases} \end{cases}$$

$$\kappa_0,\ \alpha,\ \beta$$

$$\tilde{\varepsilon} := \frac{k-1}{2k(1-2\nu)} I_1(\boldsymbol{\varepsilon}) + \frac{k-1}{2k(1-2\nu)}\sqrt{I_1^2(\boldsymbol{\varepsilon}) + \frac{12k}{(k-1)^2}\left(\frac{1-2\nu}{1-\nu}\right)^2 J_2(\boldsymbol{\varepsilon})}$$

$$k,\ c$$

$$\{k,\ c,\ \kappa_0,\ \alpha,\ \beta\} \ + \ \text{B.C.}$$

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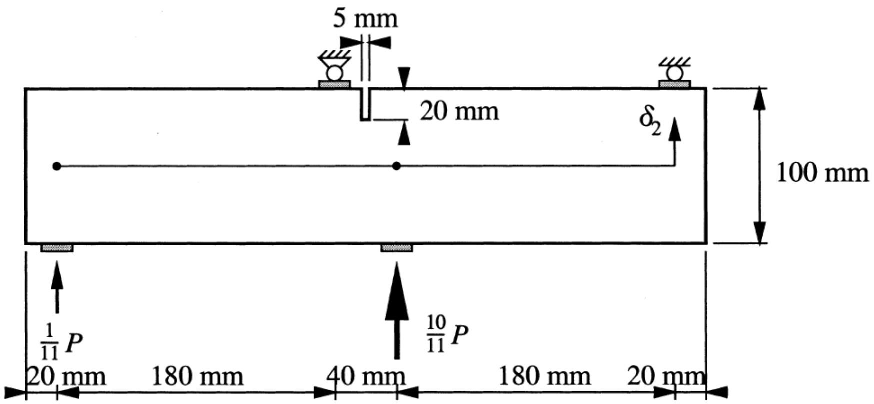
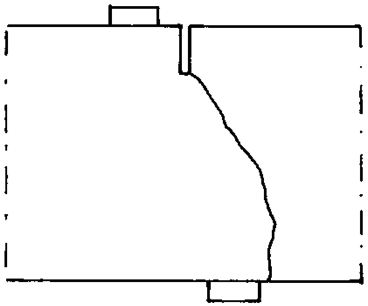


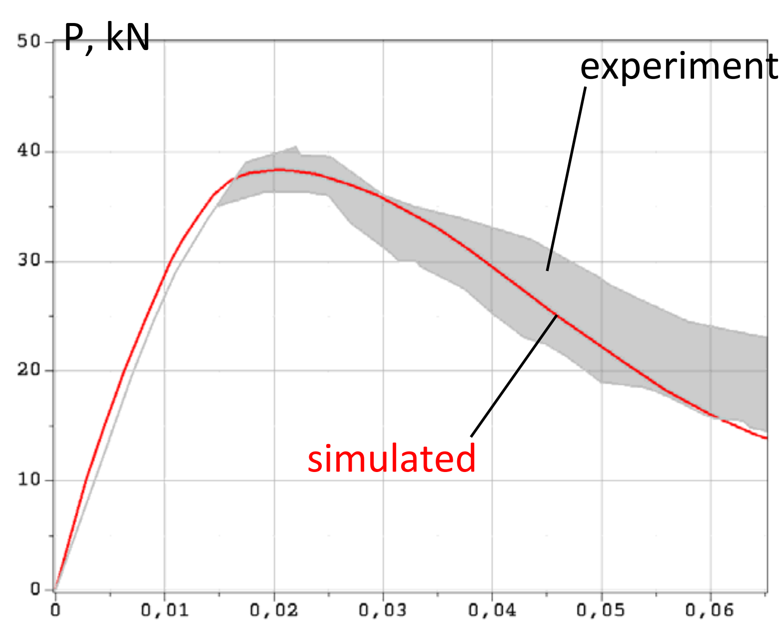
B.C. rotating supports

- plane-stress
- calibrated set:

$$\alpha := 0.96$$
$$\beta := 100$$
$$\kappa_0 := 6 \cdot 10^{-5}$$
$$k := 15$$
$$c := 1 \cdot 10^{-6}\ \text{m}^2$$

Schlangen (1993):





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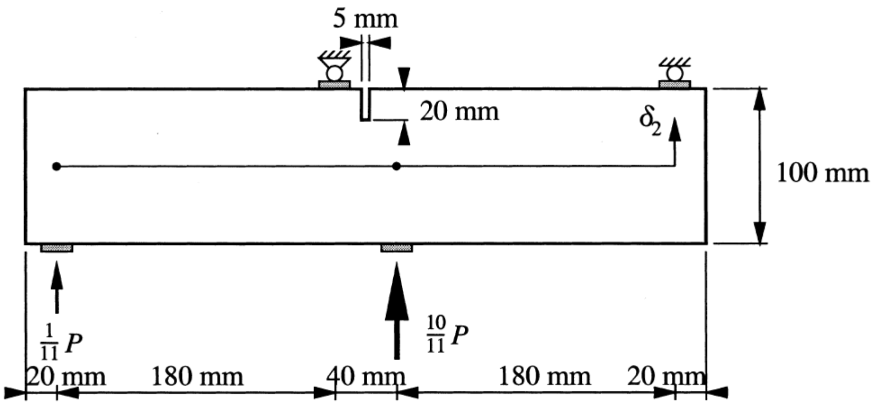

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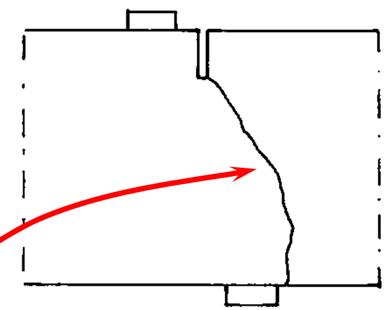
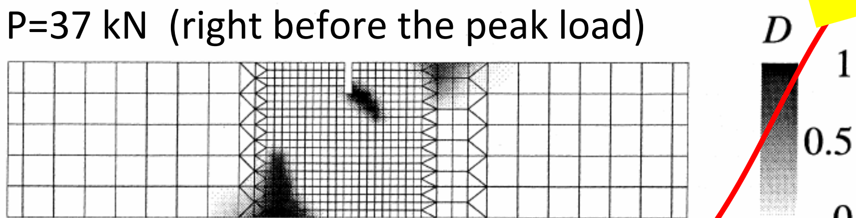
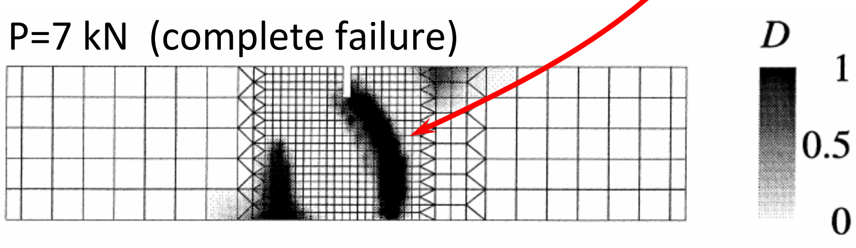


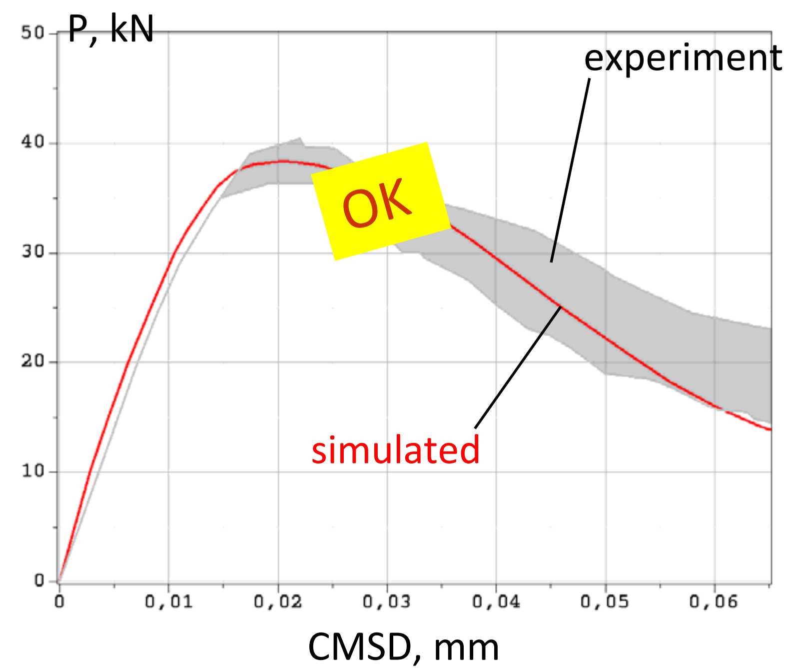
OK

P=37 kN (right before the peak load)



P=7 kN (complete failure)





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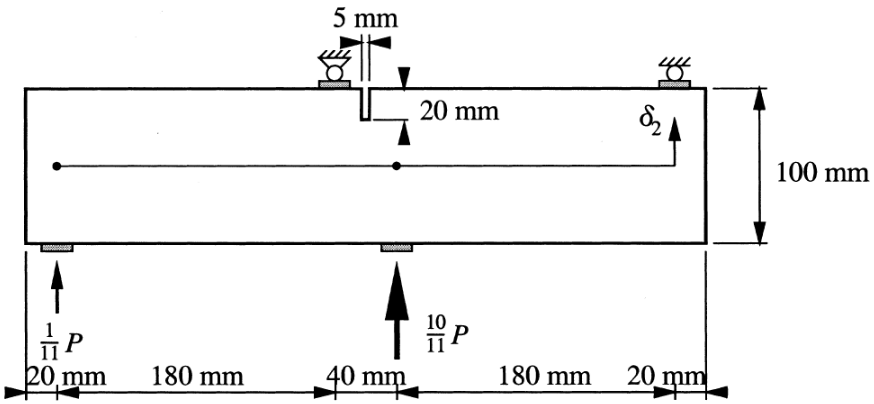

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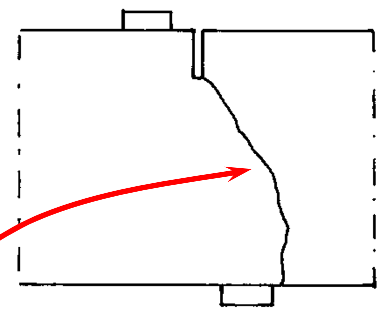

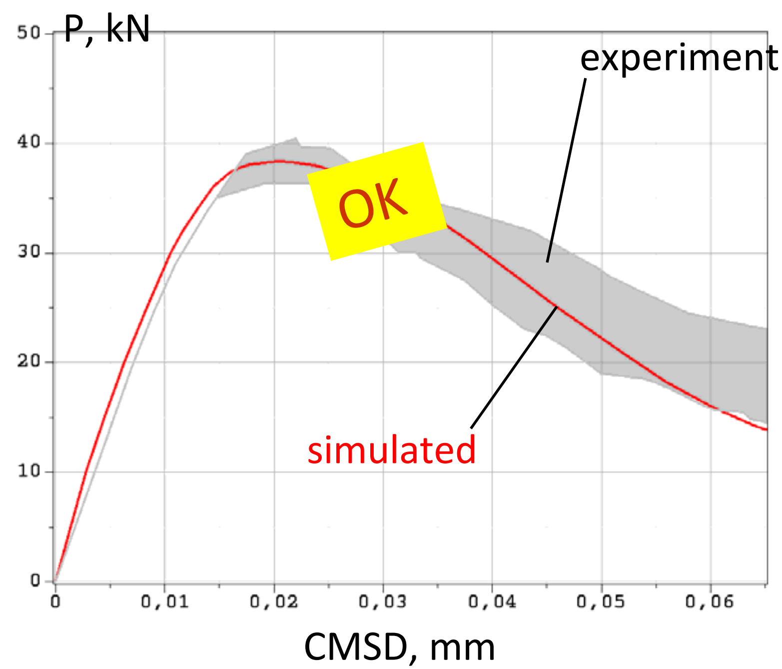

OK

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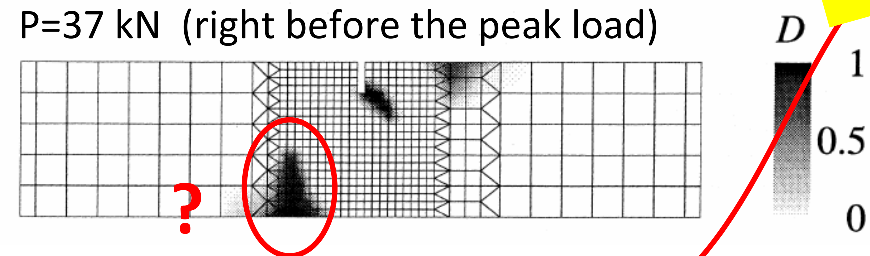

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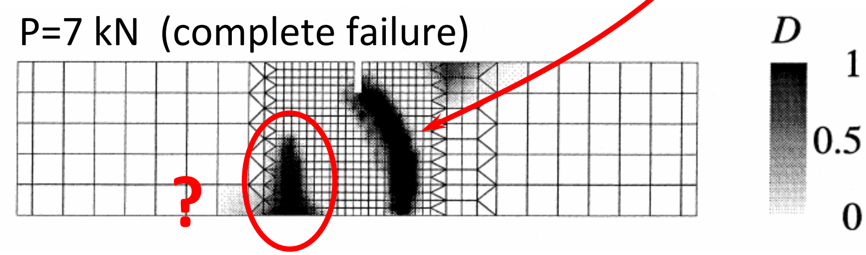

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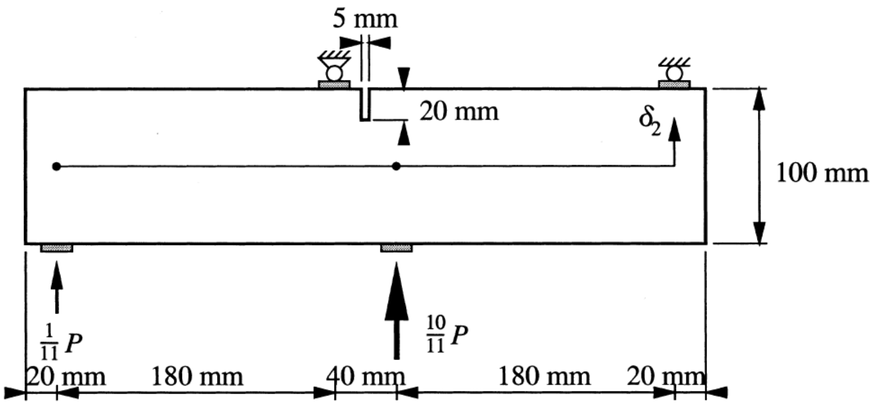

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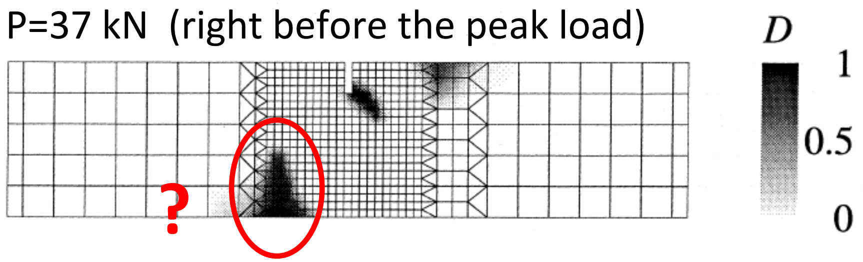

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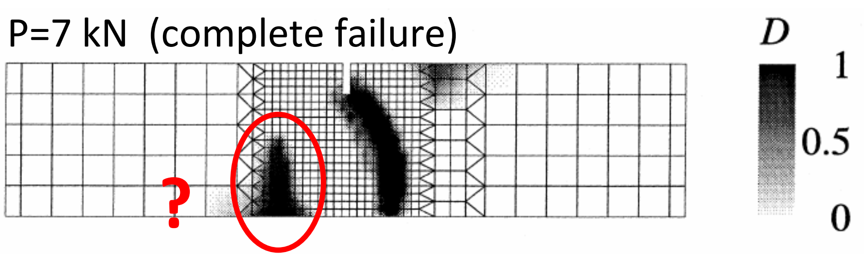
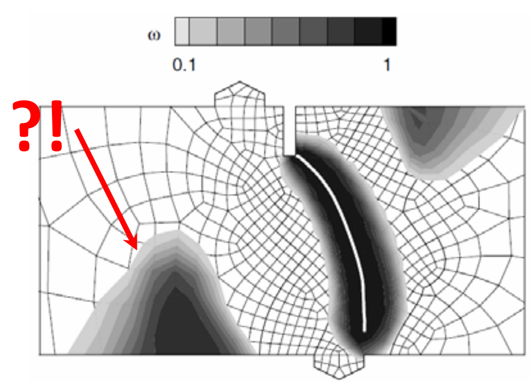


Simone *et al*. (2003):



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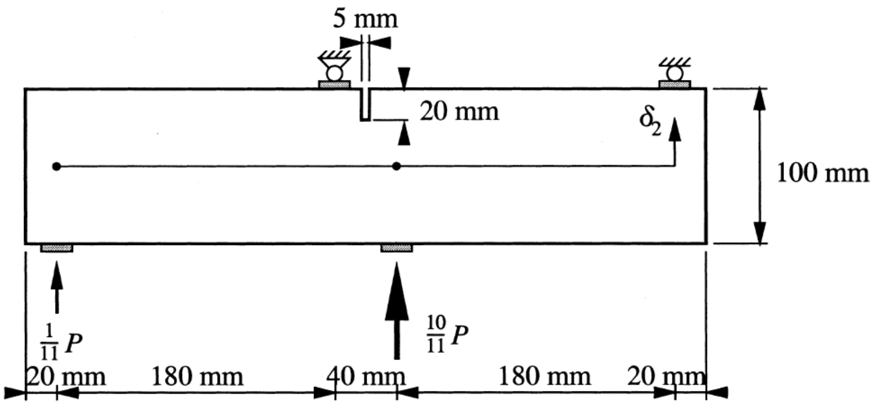
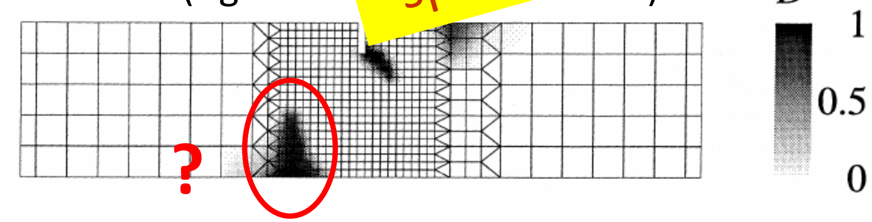


"spurious" damage zone – due to non-optimal mesh?

P=37 kN (right befo… …k load)



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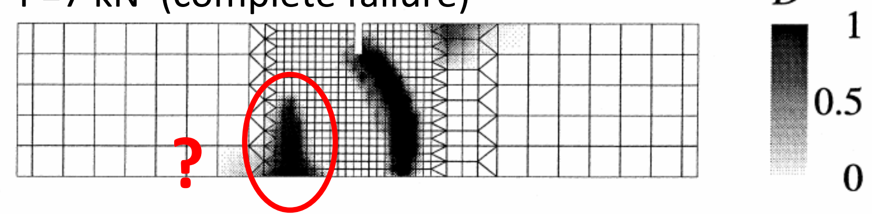

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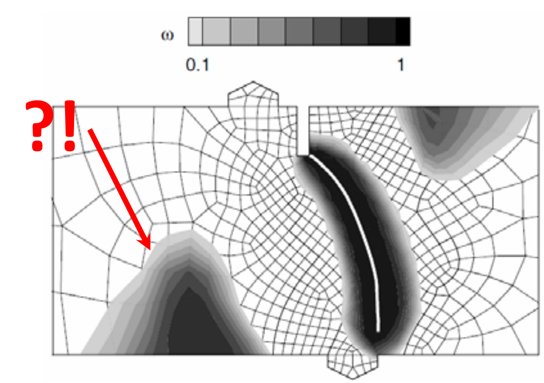
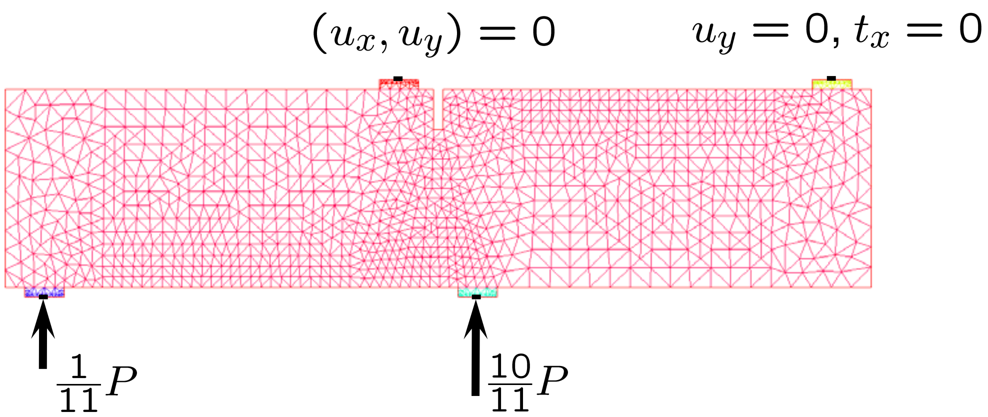
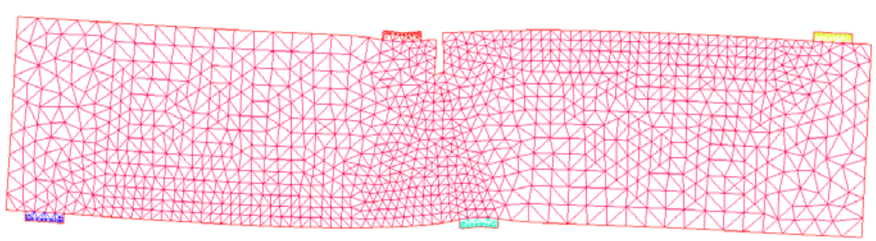


Error-controlled adaptive simulations



B.C. rotating supports



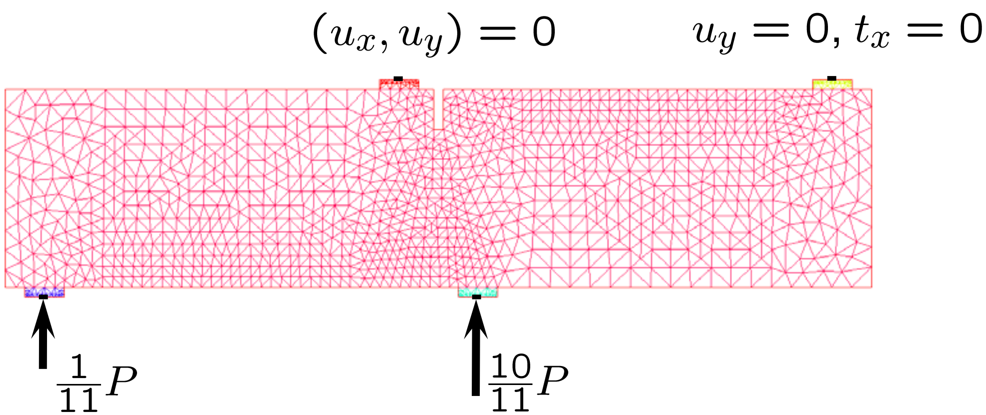
deformation (exaggerated)

$$
\begin{cases}
\boldsymbol{\sigma} = (1 - D(\bar{\varepsilon}))\,\mathbb{C} : \boldsymbol{\varepsilon} \quad \text{in} \quad \Omega \\
\begin{cases}
-c\nabla^2\bar{\varepsilon} + \bar{\varepsilon} = \tilde{\varepsilon}(\boldsymbol{\varepsilon}) \quad \text{in} \quad \Omega \\
\nabla\bar{\varepsilon} \cdot \boldsymbol{n} = 0 \quad \text{on} \quad \partial\Omega
\end{cases}
\end{cases}
$$

Let's give a try with adopted:

- $\alpha := 0.96$ $\quad k := 15$
  $\beta := 100$ $\quad c := 1 \cdot 10^{-6}\ \text{m}^2$
  $\kappa_0 := 6 \cdot 10^{-5}$ $\quad$ plane-stress

Error-controlled adaptive simulations



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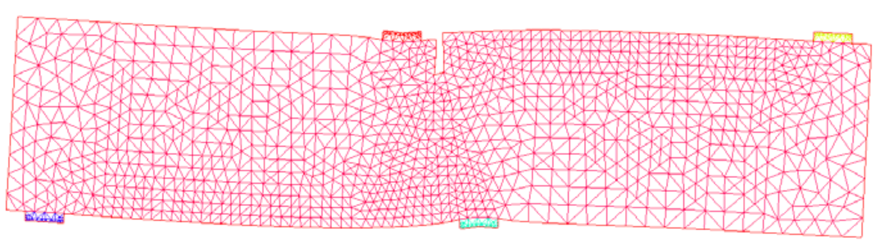

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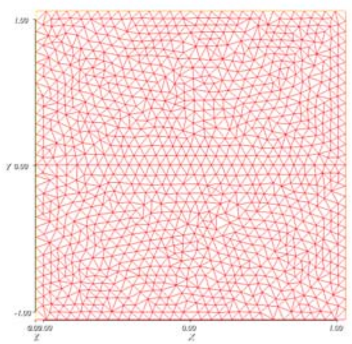
and, in our case,

- P1-triangles for both variables $(\boldsymbol{u}, \bar{\varepsilon})$
- Mesh adaptivity

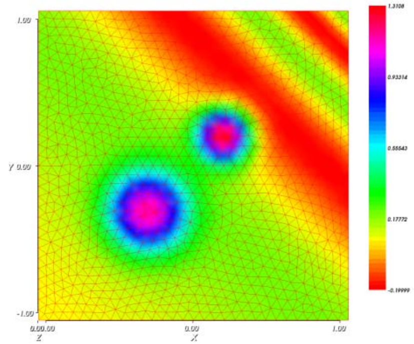
Intermezzo: mesh adaptivity (MA)

$$q(x,y) = Ae^{(-Br_1^2)} + Ce^{(-Dr_2^2)} + E\sin(2e^{x+y}) \quad \text{in} \quad [-1,1]^2$$

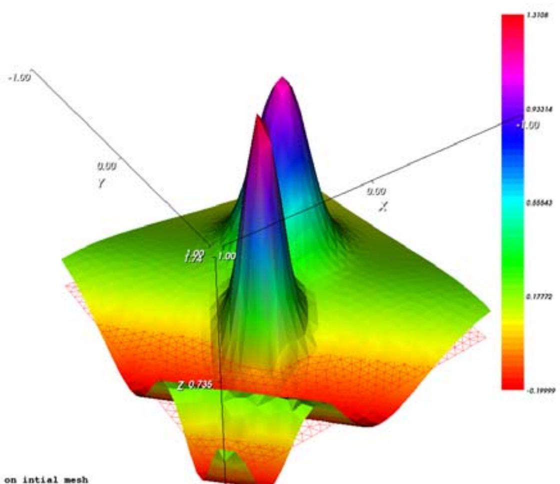
$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}, \quad i = 1,2$$



Initial mesh



q on intial mesh



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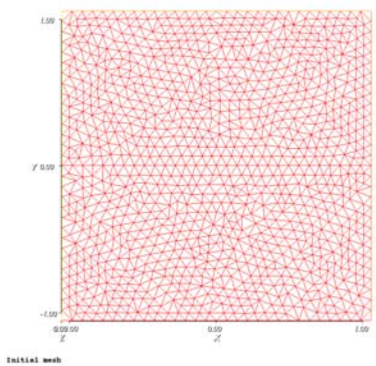

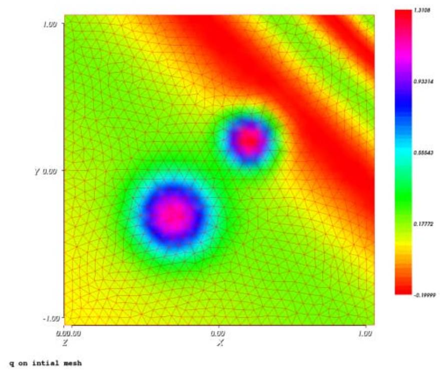

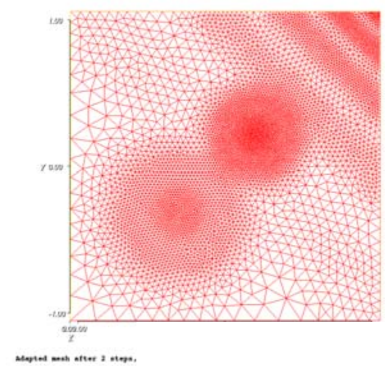

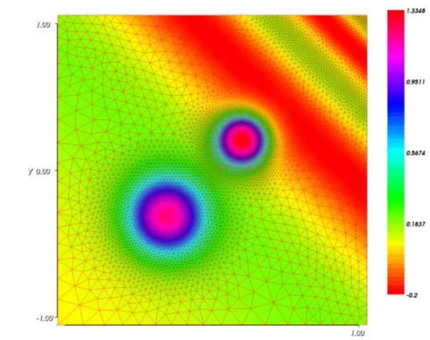

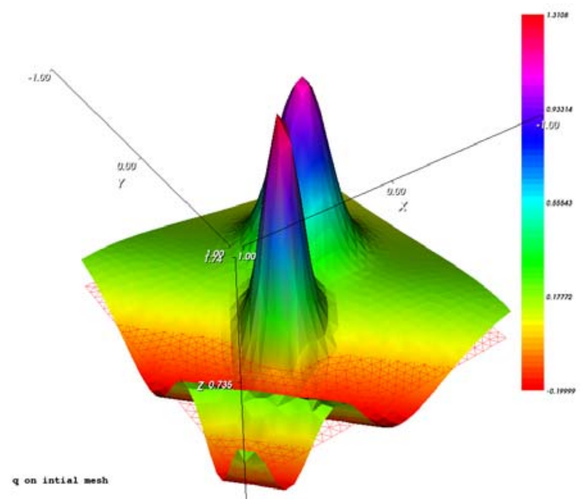

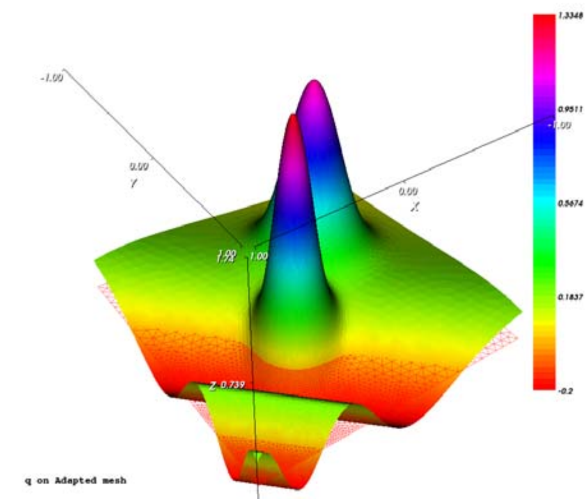

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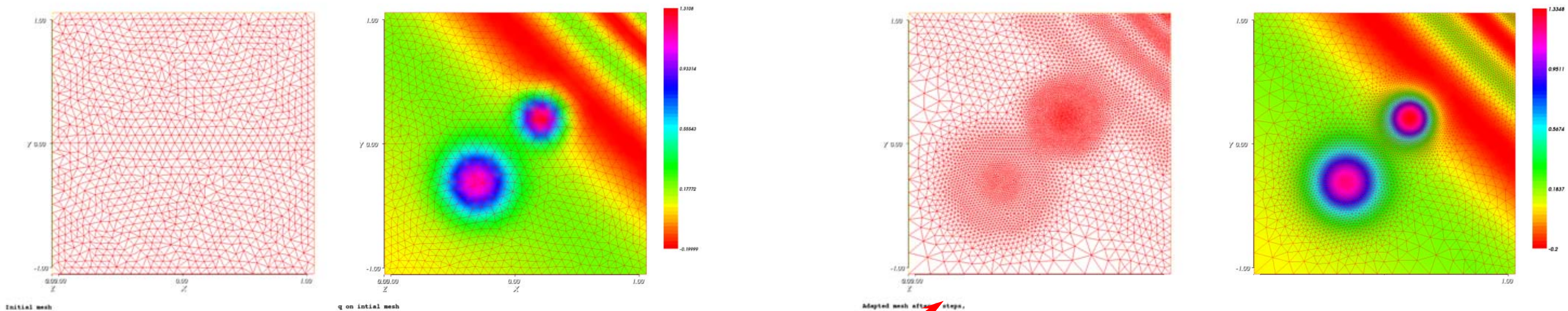
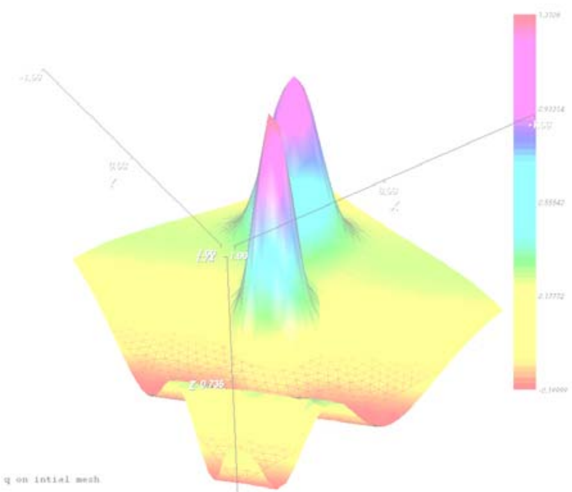
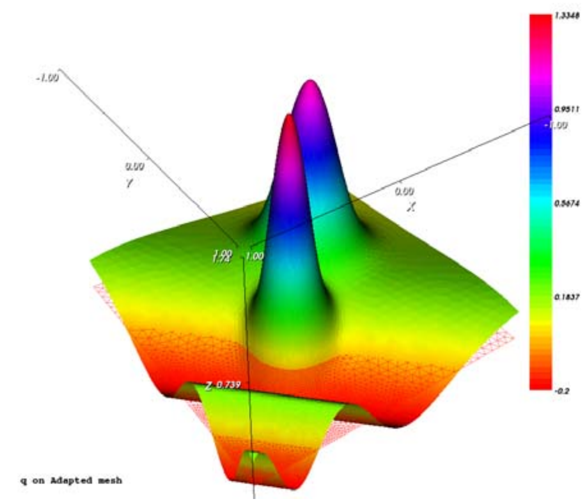
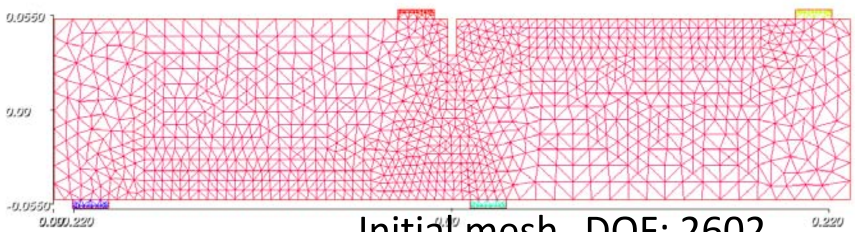




MA driven by
a priori known
prop. of a function
(e.g. steep grad.)



Intermezzo: error-controlled MA



Initial mesh, DOF: 2602

$$\begin{cases} \boldsymbol{\sigma} = (1 - D(\bar{\varepsilon}))\,\mathbb{C} : \boldsymbol{\varepsilon} \quad \text{in} \quad \Omega \\ \begin{cases} -c\nabla^2\bar{\varepsilon} + \bar{\varepsilon} = \tilde{\varepsilon}(\boldsymbol{\varepsilon}) \quad \text{in} \quad \Omega \\ \nabla\bar{\varepsilon} \cdot \boldsymbol{n} = 0 \quad \text{on} \quad \partial\Omega \end{cases} \end{cases}$$

$$(\boldsymbol{u}^h, \bar{\varepsilon}^h)$$

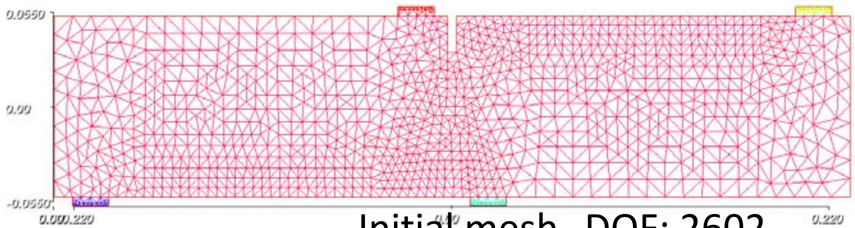
Discretization Error:

unknown

$$|||\boldsymbol{u} - \boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon} - \bar{\varepsilon}^h||^*_\Omega$$

computed

Intermezzo: error-controlled MA



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$$(\boldsymbol{u}^h, \bar{\varepsilon}^h)$$

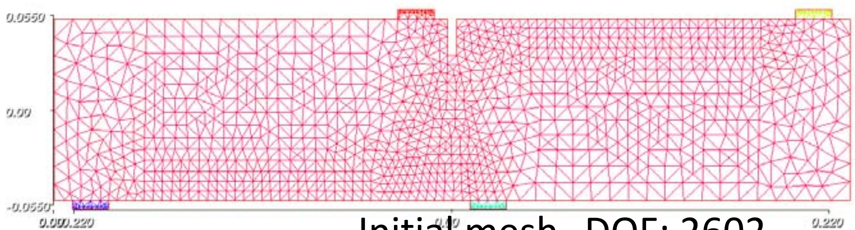
unknown

$$|||\boldsymbol{u} - \boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon} - \bar{\varepsilon}^h||^*_\Omega \leq \text{UB}(\boldsymbol{u}^h, \bar{\varepsilon}^h; \text{BF}, \text{NeumBC})$$

computed

$$\text{UB} := \left(\sum_T {\eta_T}^2 \right)^{\frac{1}{2}}$$

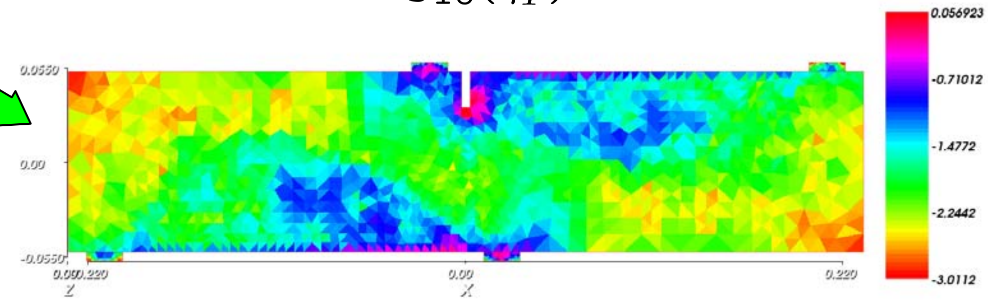
Intermezzo: error-controlled MA



Initial mesh, DOF: 2602

local error indicators

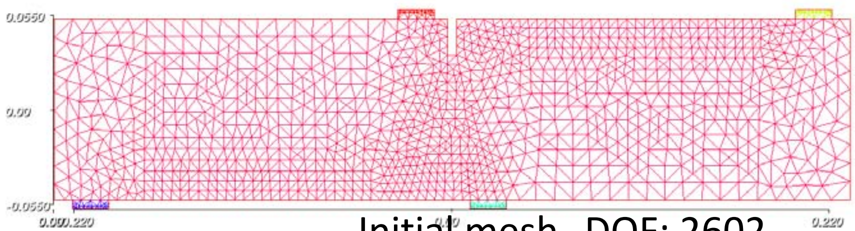
$\log_{10}(\eta_T)$



$$|||\boldsymbol{u} - \boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon} - \bar{\varepsilon}^h||^*_\Omega \leq \mathsf{UB}(\boldsymbol{u}^h, \bar{\varepsilon}^h; \mathsf{BF}, \mathsf{NeumBC})$$

$$\mathsf{UB} := \left(\sum_T {\eta_T}^2\right)^{\frac{1}{2}}$$

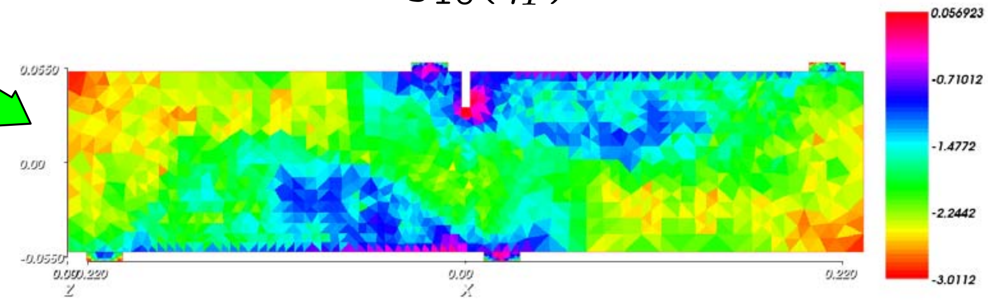
Intermezzo: error-controlled MA



Initial mesh, DOF: 2602

local error indicators

$\log_{10}(\eta_T)$



$$|||\boldsymbol{u}-\boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon}-\bar{\varepsilon}^h||^*_\Omega \leq \text{UB}(\boldsymbol{u}^h,\bar{\varepsilon}^h;\text{BF},\text{NeumBC})$$

$$\text{UB} := \left(\sum_T {\eta_T}^2\right)^{\frac{1}{2}}$$

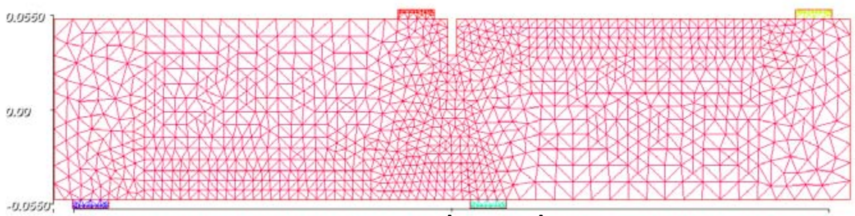
MA driven by known local errors η_T

$$h_1(\boldsymbol{x}) = \frac{h_0(\boldsymbol{x})}{f(\eta_T)}$$

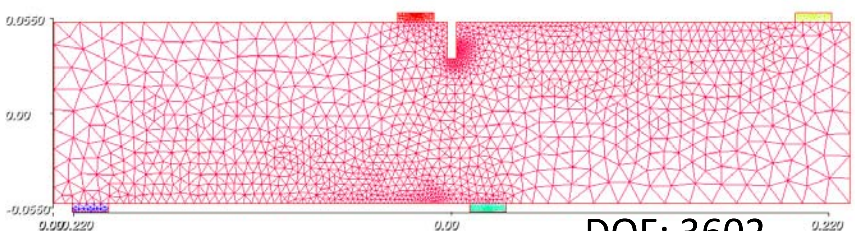
Intermezzo: error-controlled MA

local error indicators

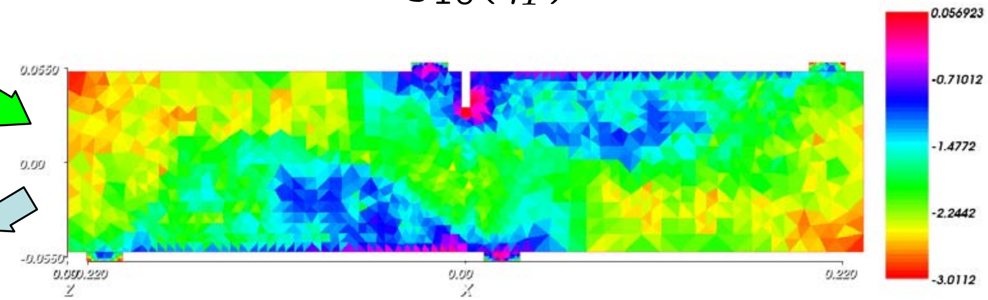
$\log_{10}(\eta_T)$



Initial mesh, DOF: 2602



DOF: 3602



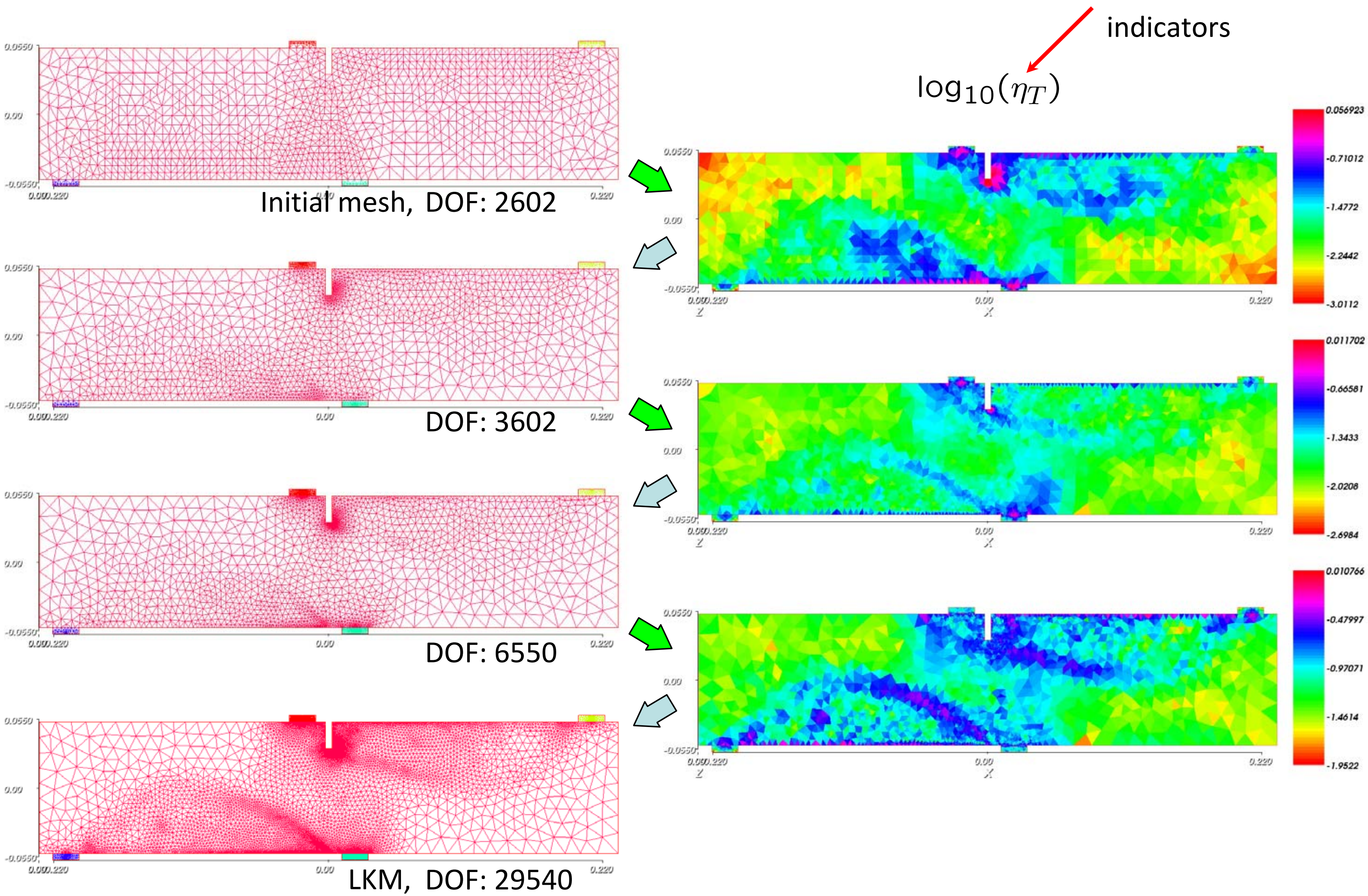
$$|||\boldsymbol{u}-\boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon}-\bar{\varepsilon}^h||^*_\Omega \leq \mathsf{UB}(\boldsymbol{u}^h, \bar{\varepsilon}^h; \mathsf{BF}, \mathsf{NeumBC})$$

$$\mathsf{UB} := \left(\sum_T {\eta_T}^2\right)^{\frac{1}{2}}$$

MA driven by known local errors η_T

$$h_1(\boldsymbol{x}) = \frac{h_0(\boldsymbol{x})}{f(\eta_T)}$$

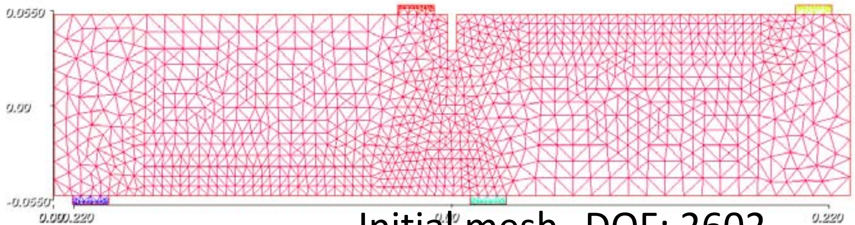
Intermezzo: error-controlled MA



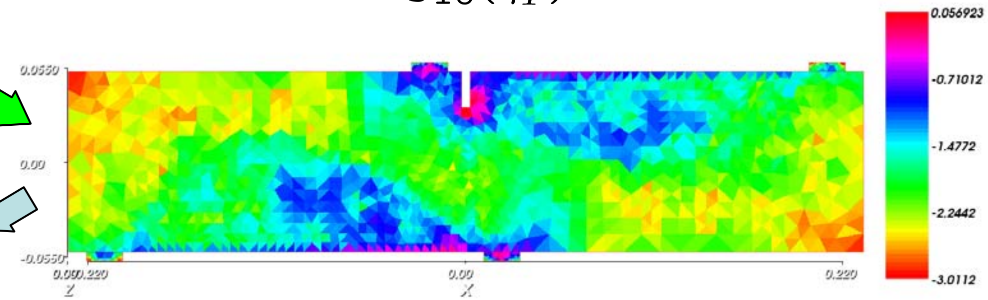
Intermezzo: error-controlled MA

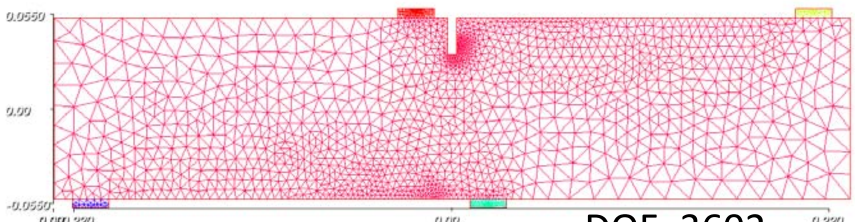
local error indicators

$\log_{10}(\eta_T)$

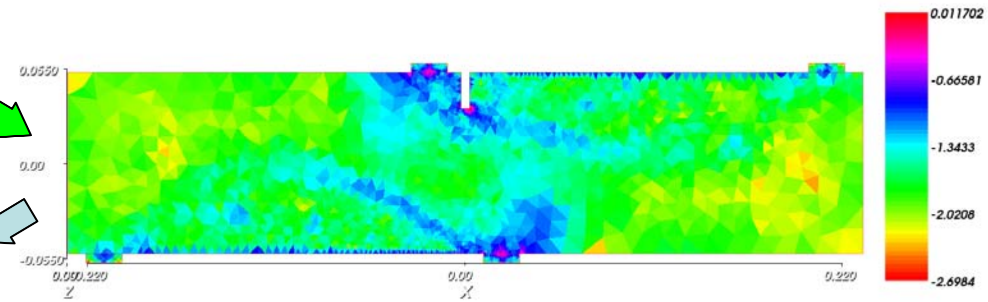


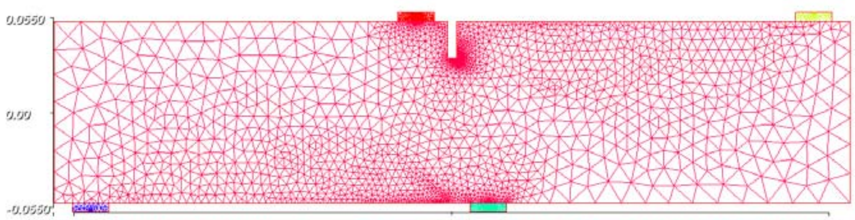
Initial mesh, DOF: 2602



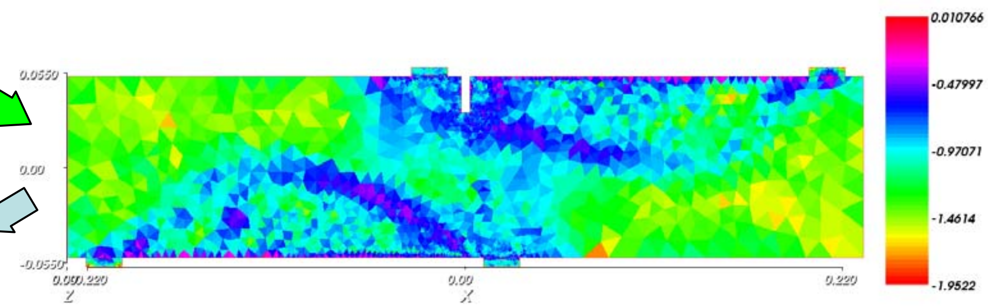


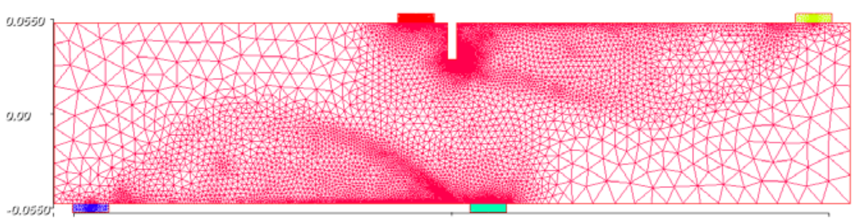
DOF: 3602





DOF: 6550





LKM, DOF: 29540

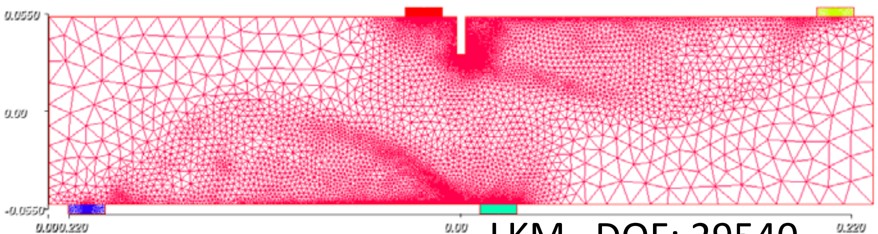
$$|||\boldsymbol{u} - \boldsymbol{u}^h|||_\Omega + ||\bar{\varepsilon} - \bar{\varepsilon}^h||^*_\Omega \leq \text{UB} \leq \text{tol}^\star$$

★ Gerasimov, Stein, Wriggers (2013), in preparation

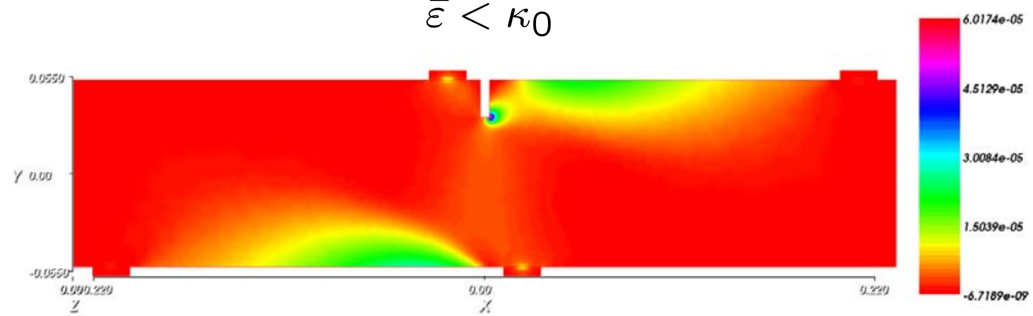
Error-controlled adaptive simulations

BEFORE damage initiation

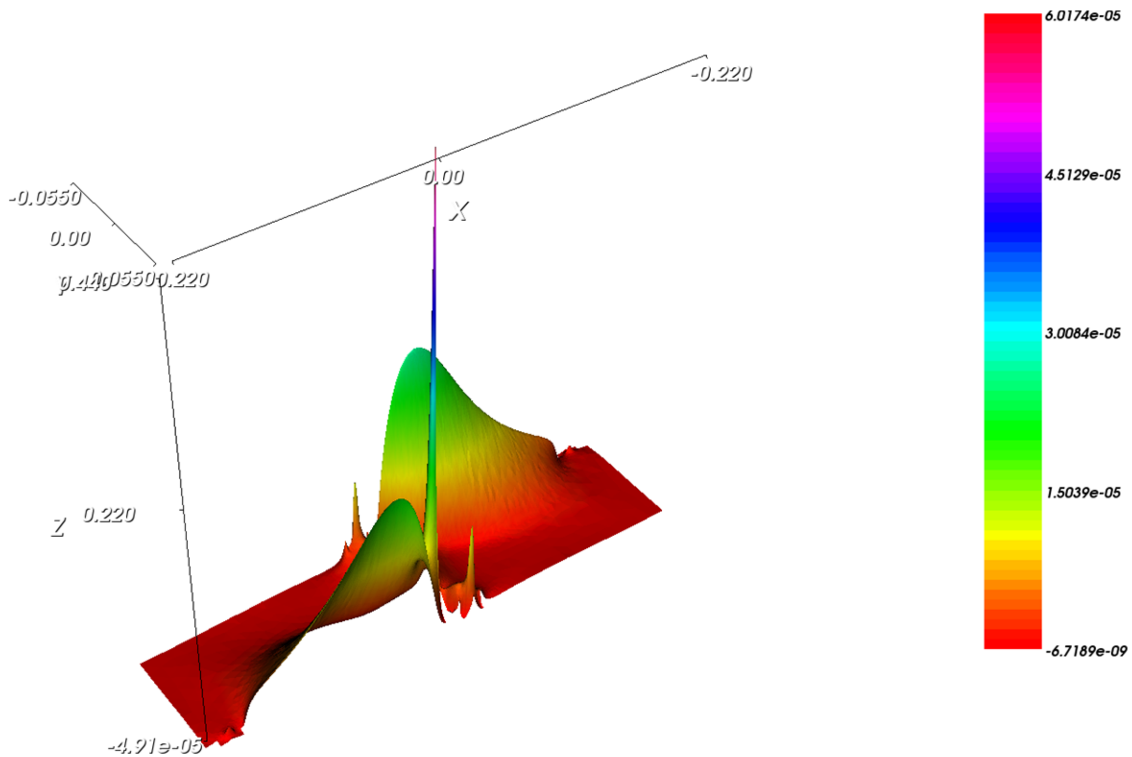
$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540



Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

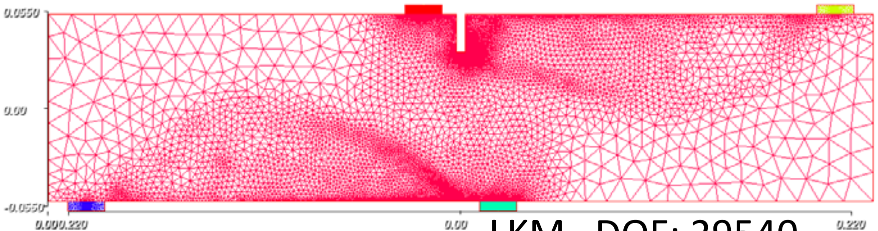


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

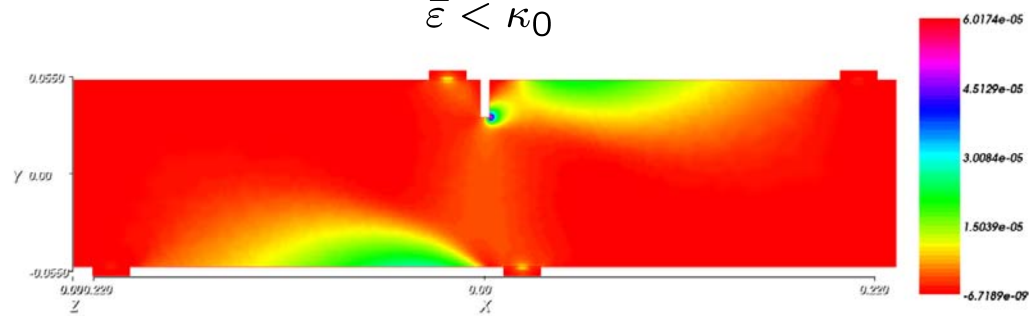
Error-controlled adaptive simulations

BEFORE damage initiation

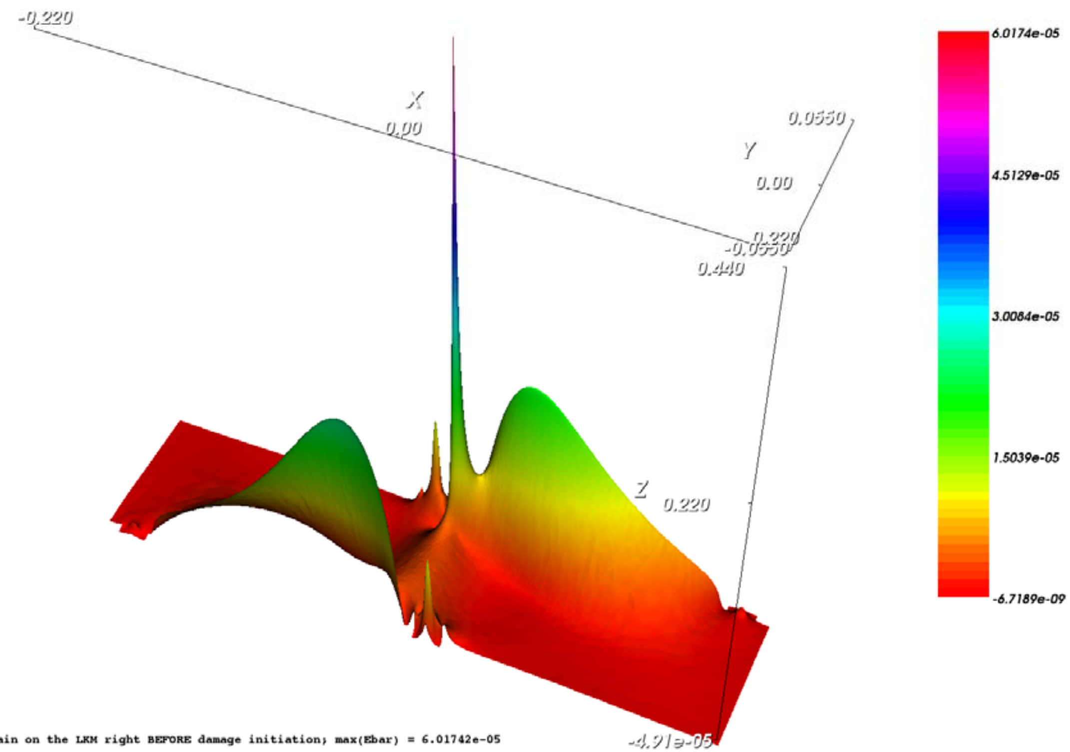
$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540



Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

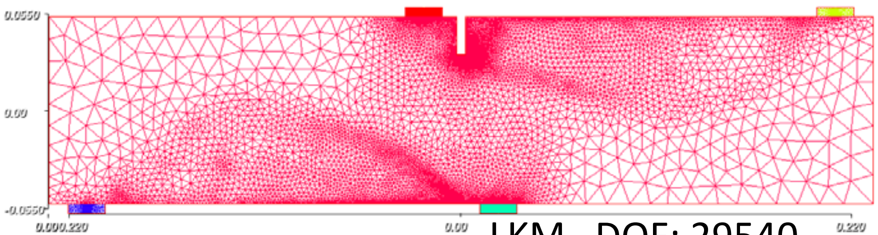


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

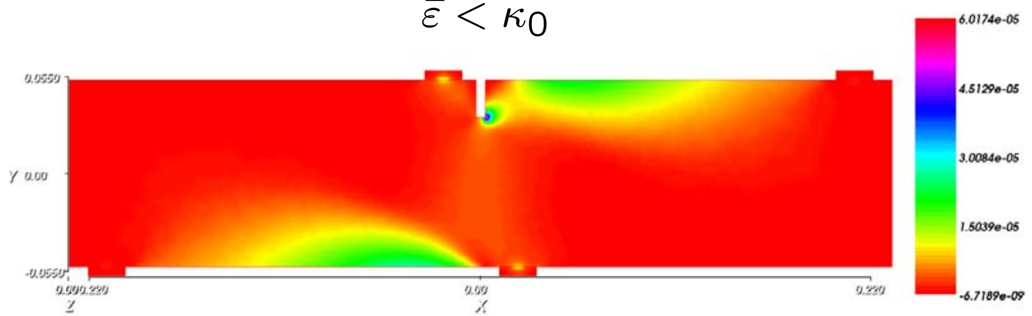
Error-controlled adaptive simulations

BEFORE damage initiation

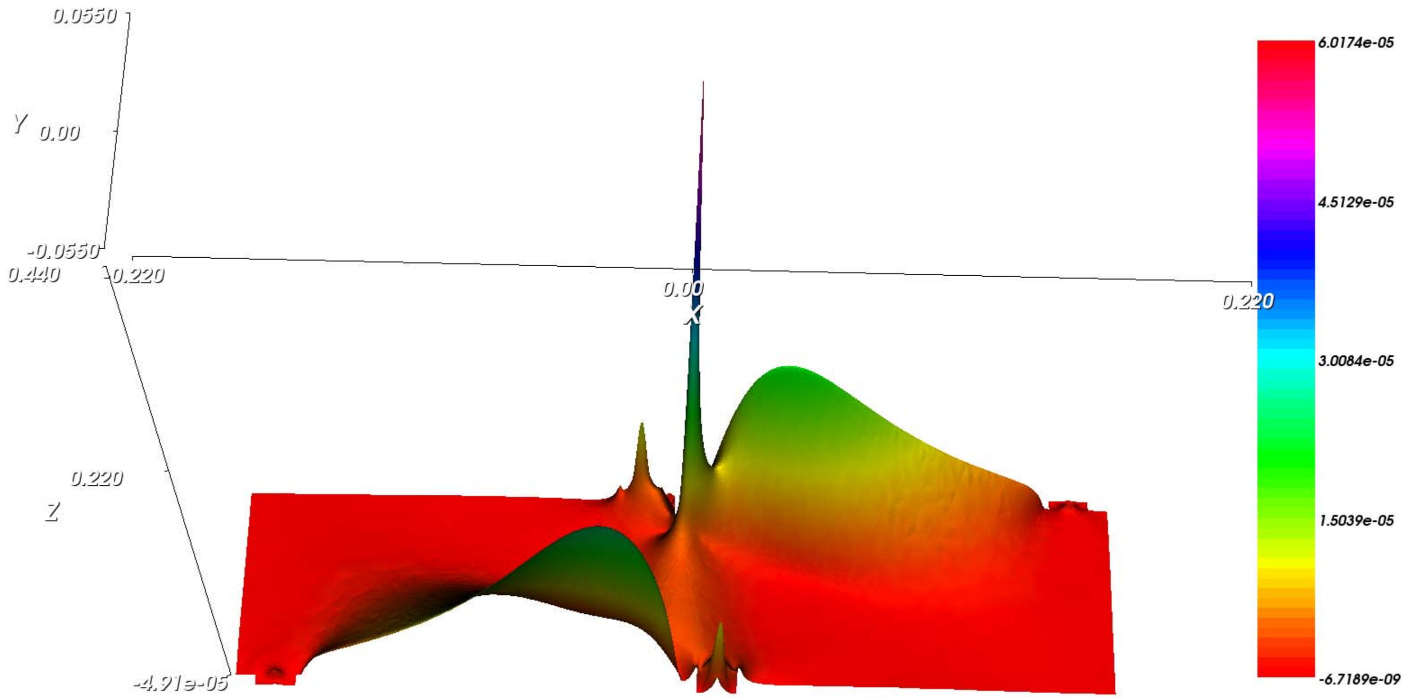
$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540



Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

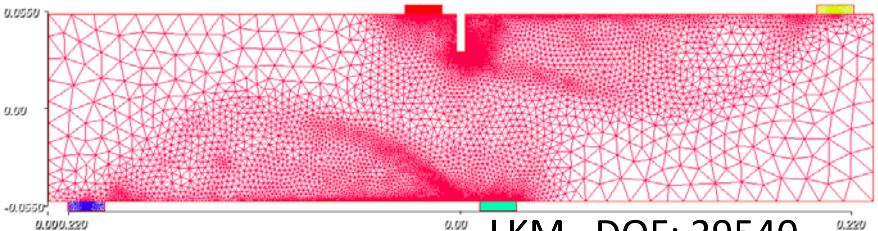


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

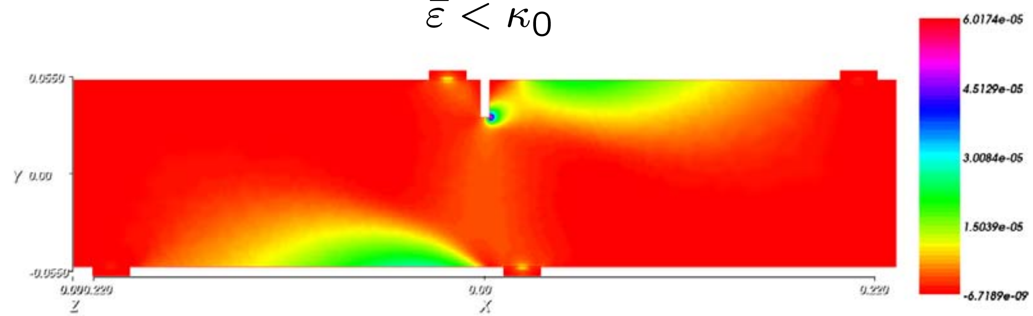
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540



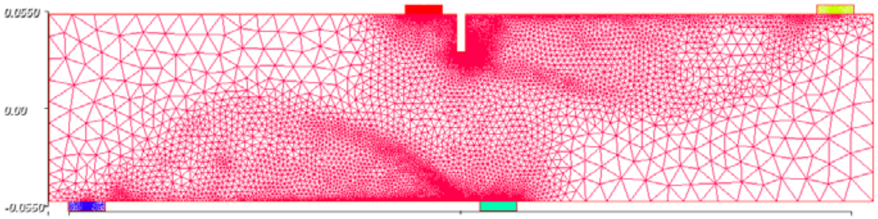
Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

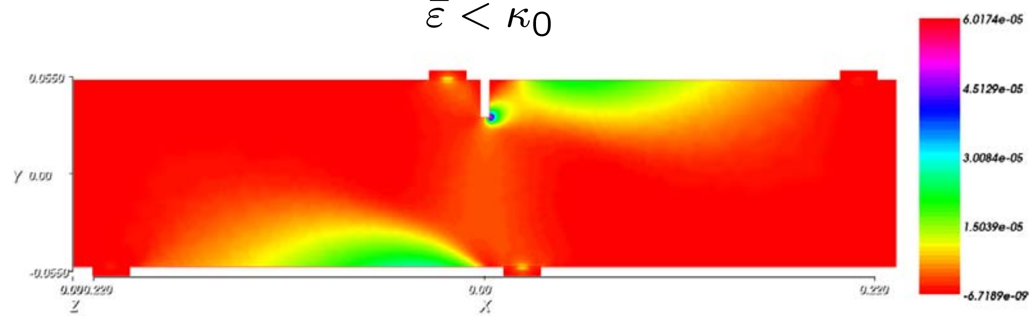
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

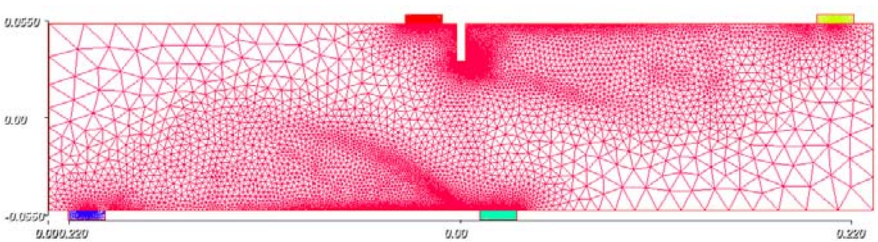


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

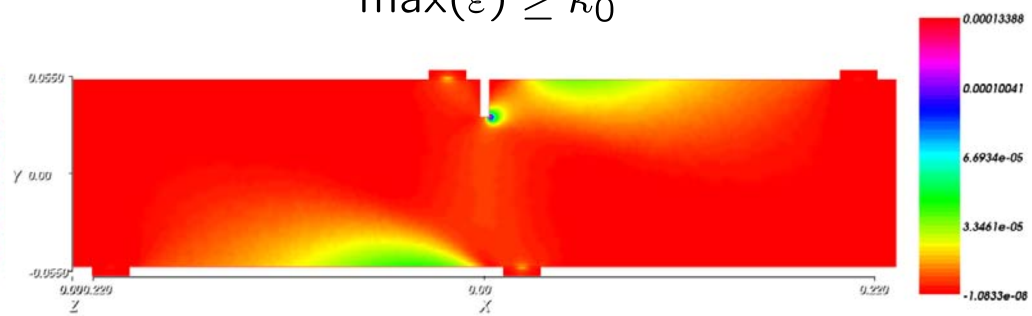
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 1

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 32130

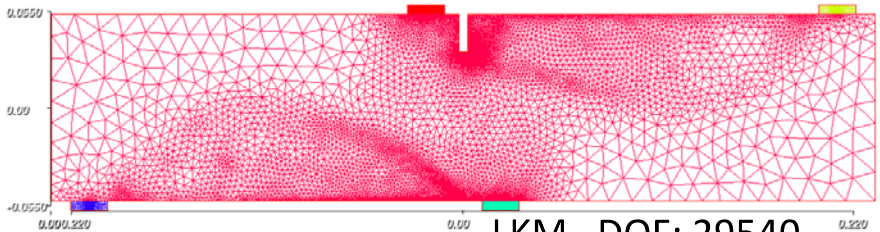


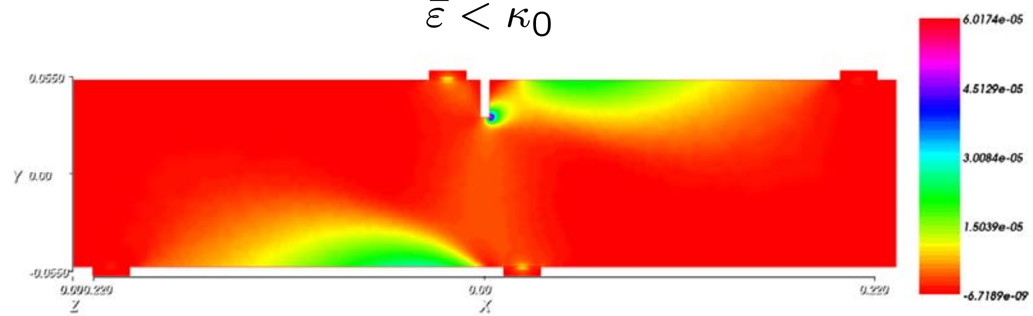
Ebar at [1,i+1]=[1,3]

Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$





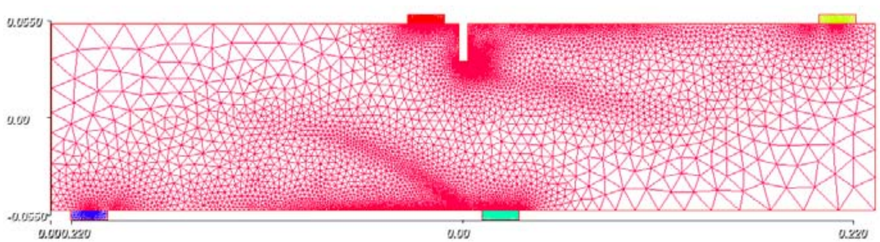
LKM, DOF: 29540

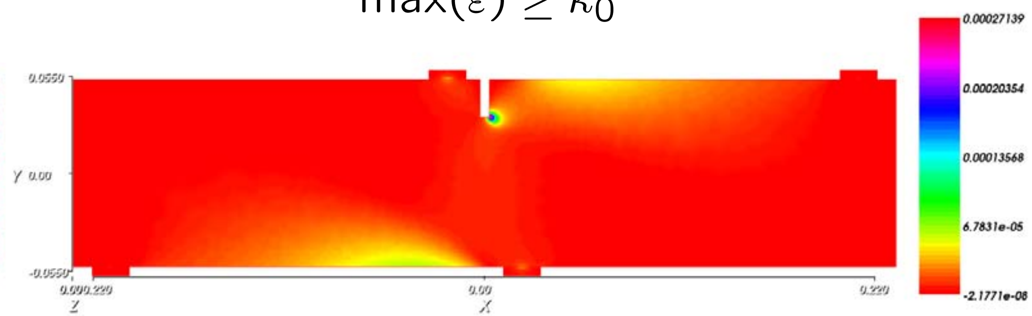
Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 2

$\max(\bar{\varepsilon}) \geq \kappa_0$





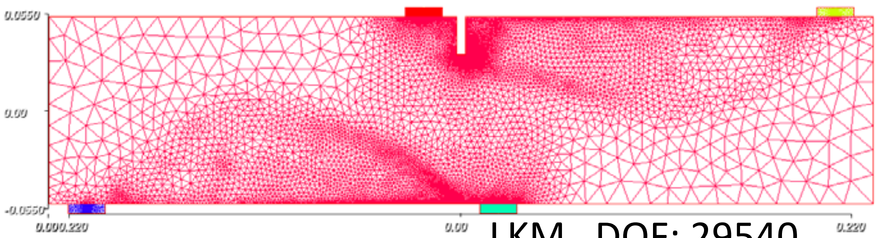
DOF: 27802

Ebar at [1,i+1]=[2,3]

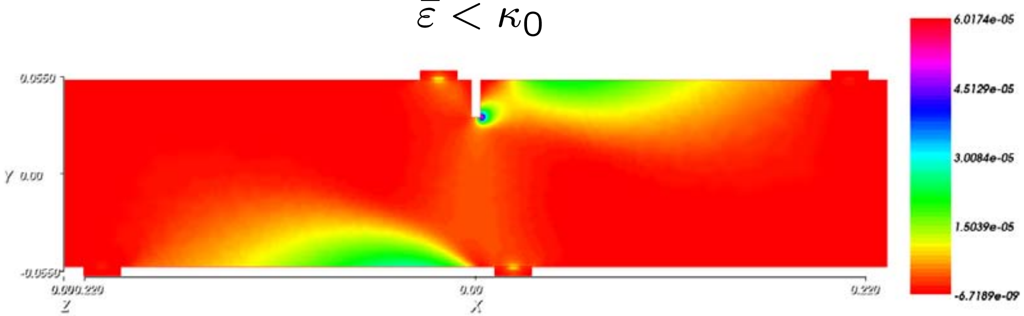
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

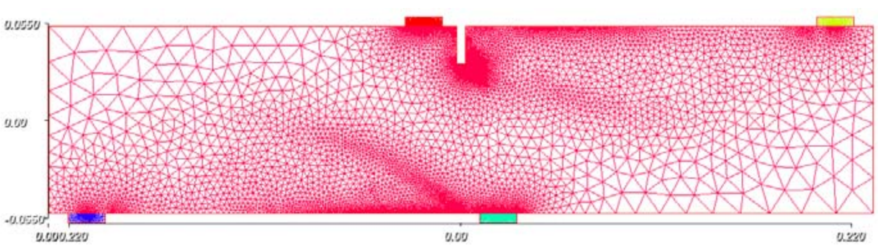


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

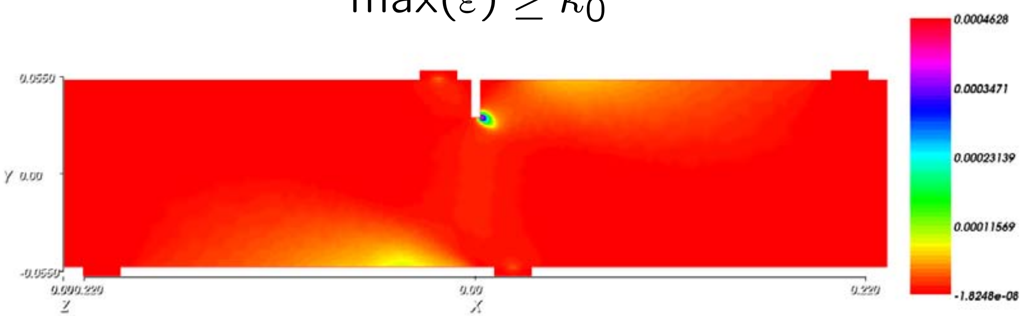
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution, goal to keep the amount of DOF ~ 30 k

Loading Step 3

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 30168

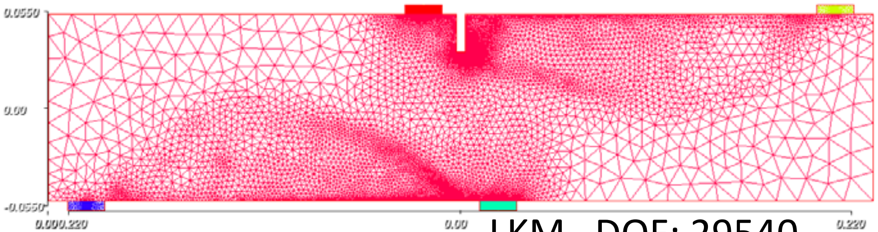


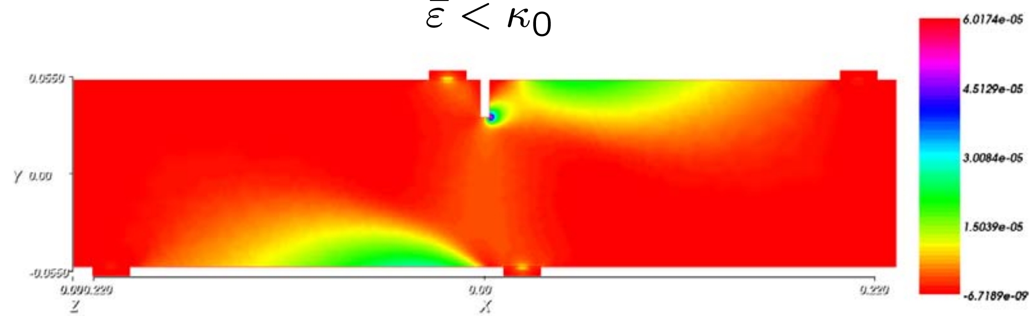
Ebar at [1,i+1]=[3,4]

Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$





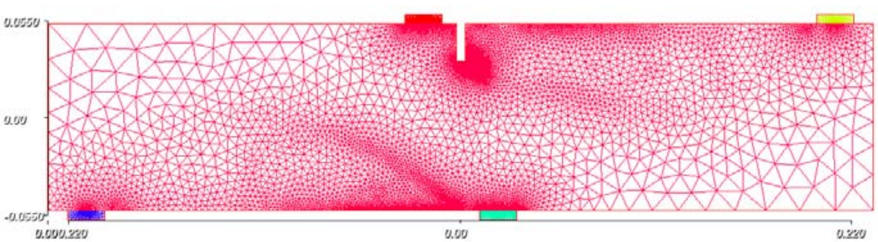
LKM, DOF: 29540

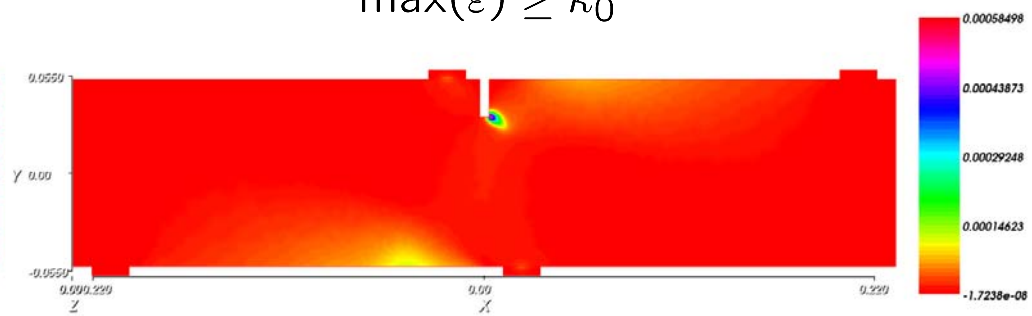
Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 4

$\max(\bar{\varepsilon}) \geq \kappa_0$





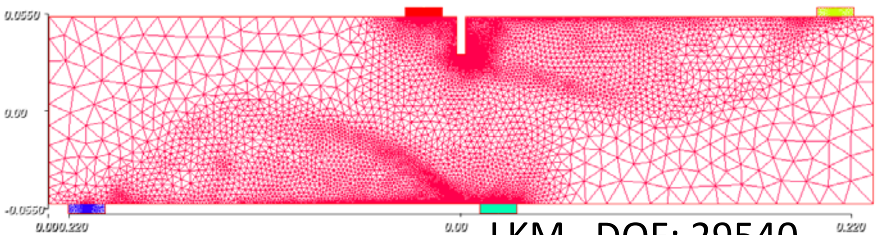
DOF: 29198

Ebar at [1,i+1]=[4,4]

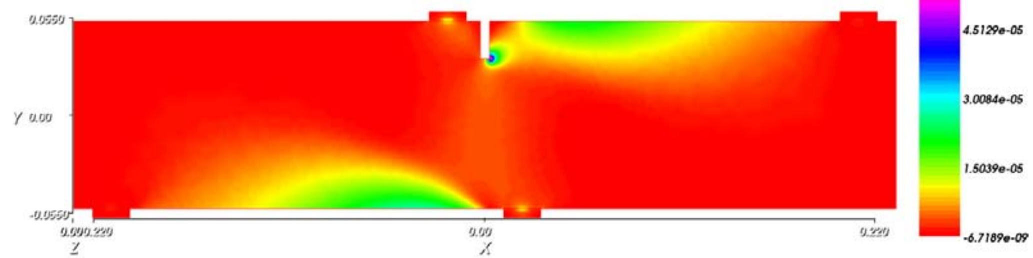
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

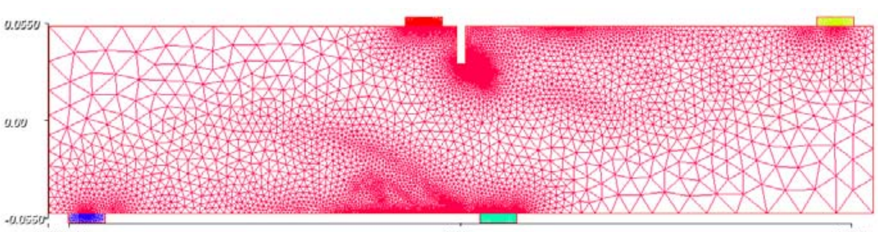


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

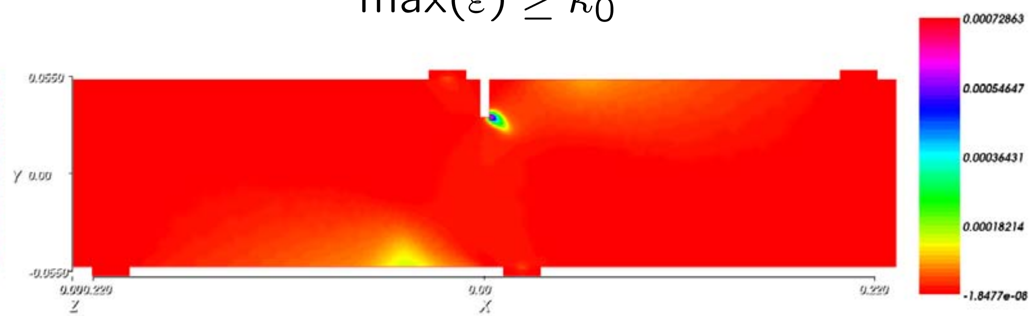
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 5

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 29186

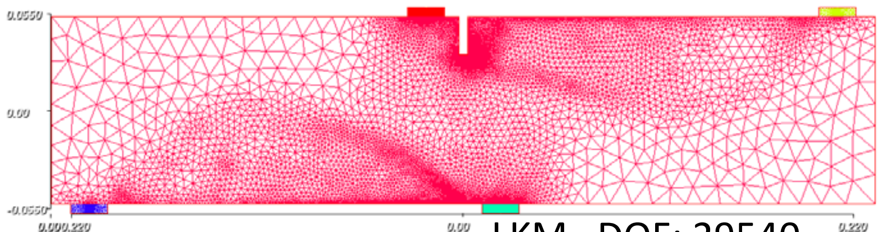


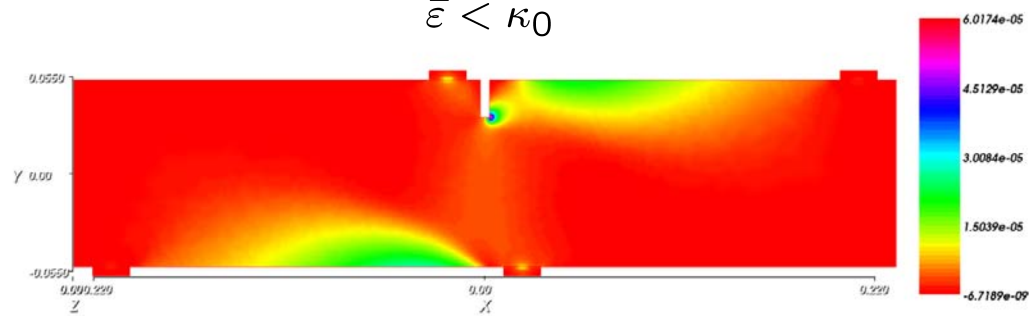
Ebar at [1,i+1]=[5,4]

Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$





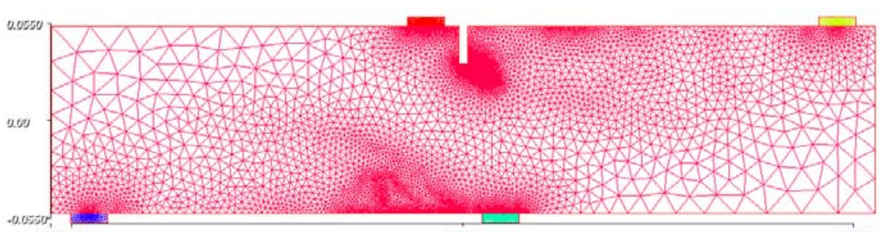
LKM, DOF: 29540

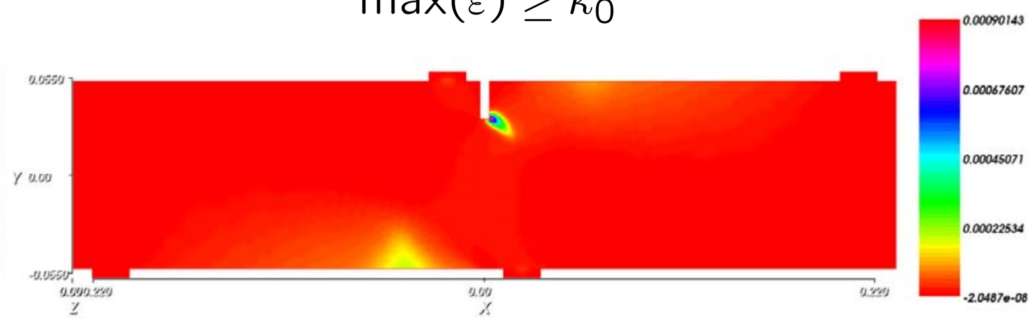
Ebar, Non-local Equiv. strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution, goal to keep the amount of DOF ~ 30 k

Loading Step 6

$\max(\bar{\varepsilon}) \geq \kappa_0$





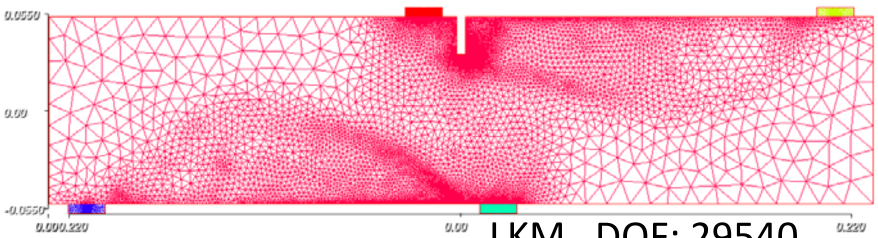
DOF: 29242

Ebar at [1,i+1]=[6,5]

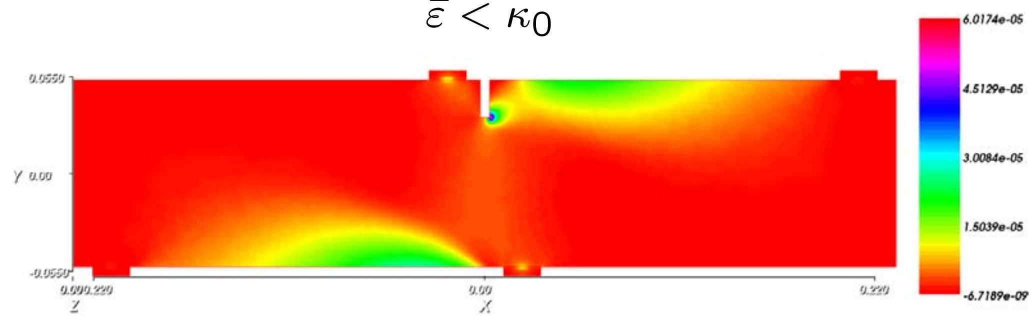
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



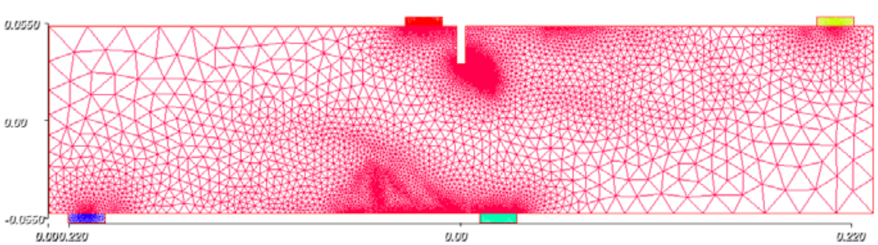
LKM, DOF: 29540



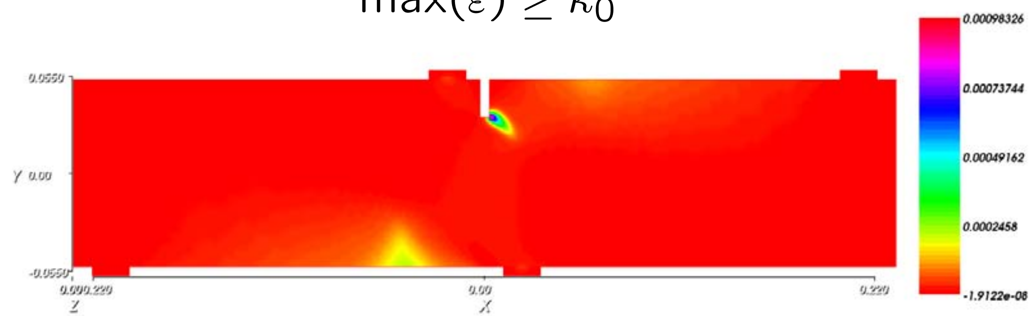
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 7

$\max(\bar{\varepsilon}) \geq \kappa_0$



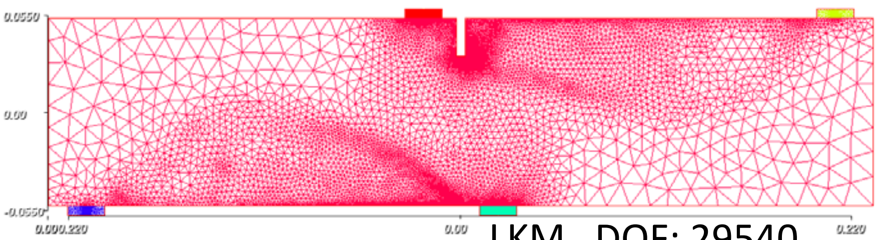
DOF: 29384



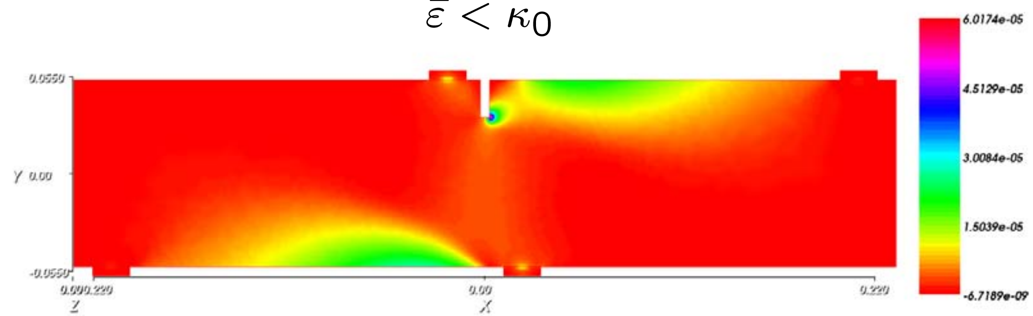
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

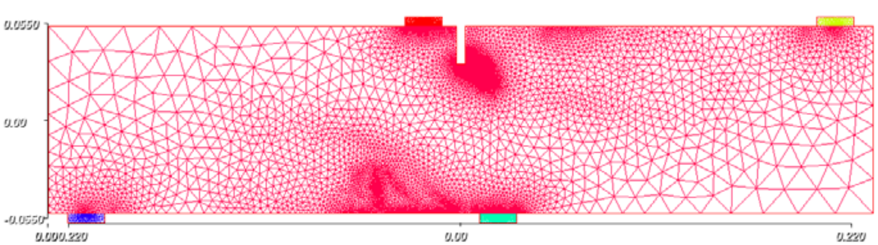


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

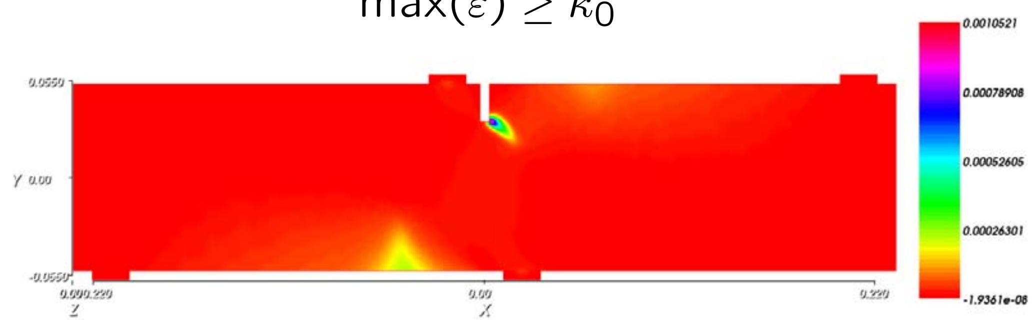
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution, goal to keep the amount of DOF ~ 30 k

Loading Step 8

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 29240

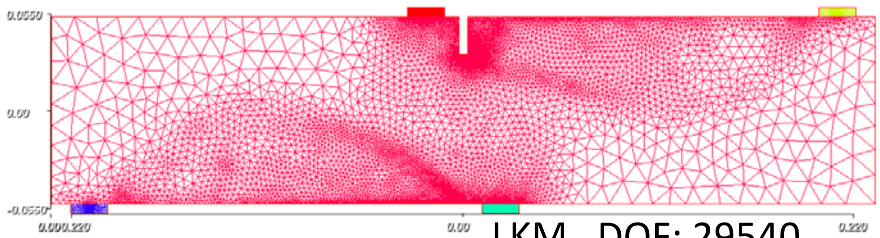


Ebar at [1,i+1]=[8,9]

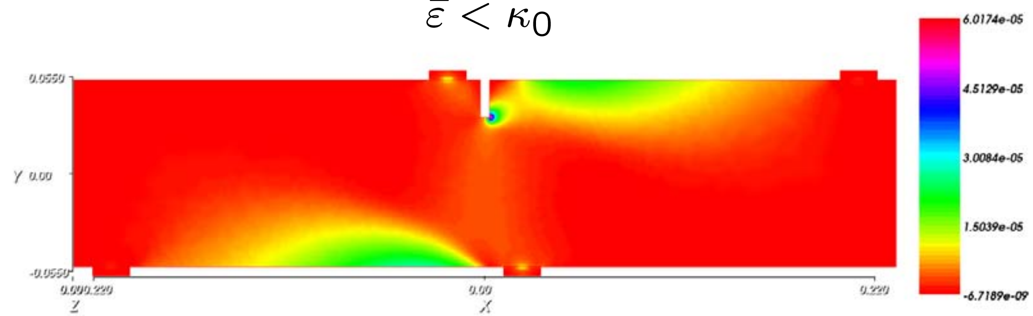
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

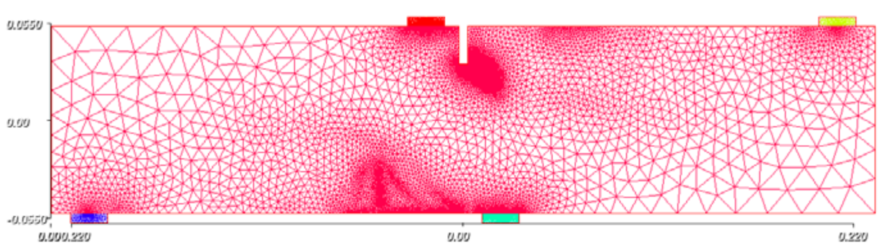


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

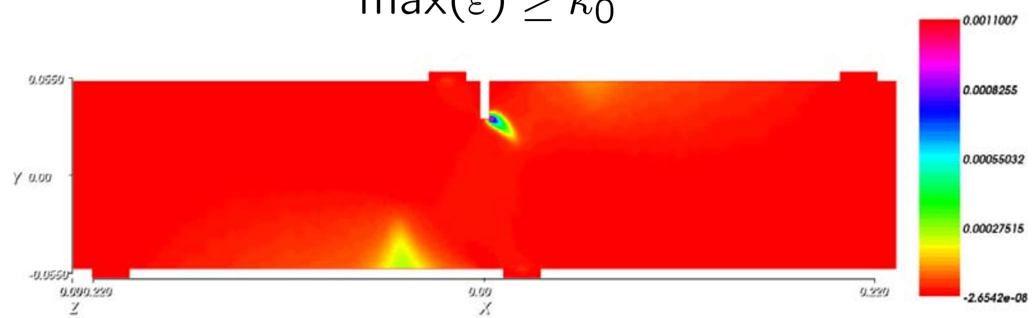
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 9

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 29438

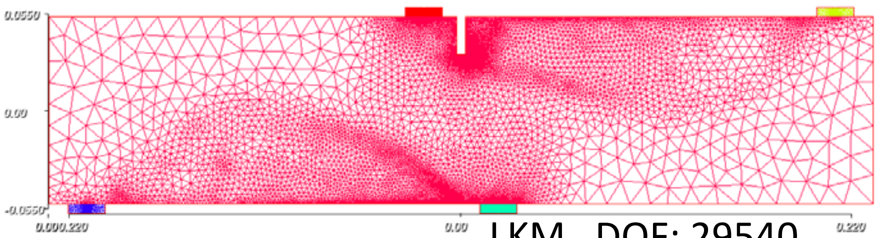


Ebar at [1,i+1]=[9,11]

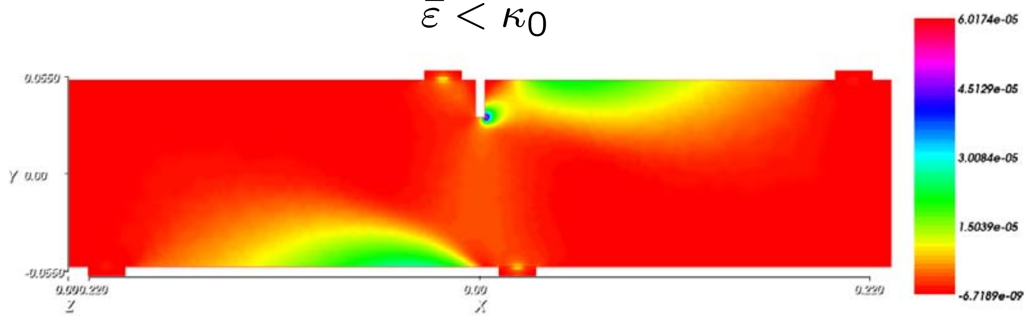
Error-controlled adaptive simulations

BEFORE damage initiation

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

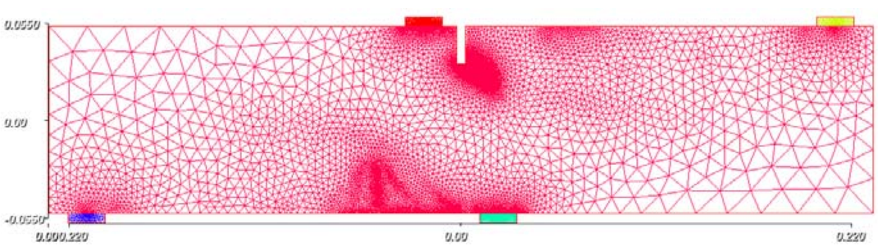


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

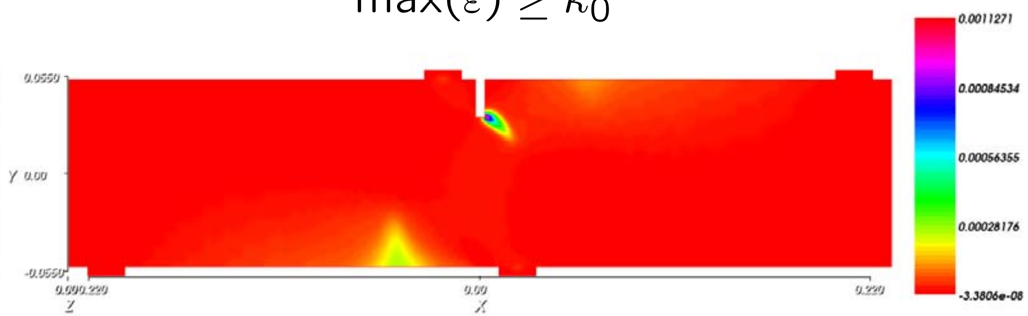
AFTER damage initiation: adaptive remeshing w.r.t. $\bar{\varepsilon}$ evolution,
goal to keep the amount of DOF ~ 30 k

Loading Step 10 (**my peak load**)

$\max(\bar{\varepsilon}) \geq \kappa_0$



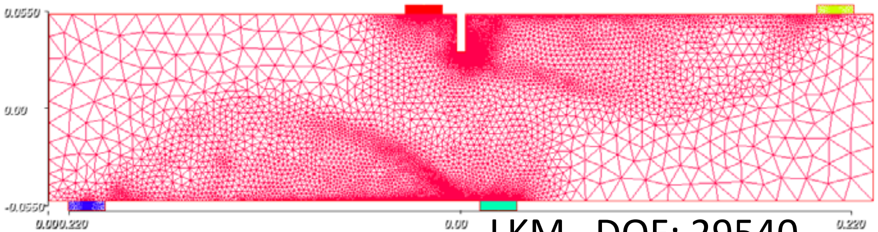
DOF: 29360

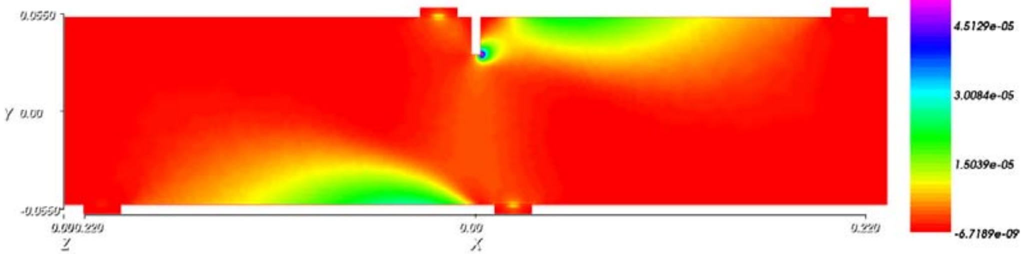


Ebar at [1,i+1]=[10,13]

Error-controlled adaptive simulations

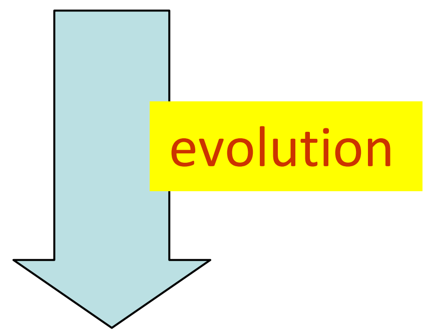
$\bar{\varepsilon} < \kappa_0$



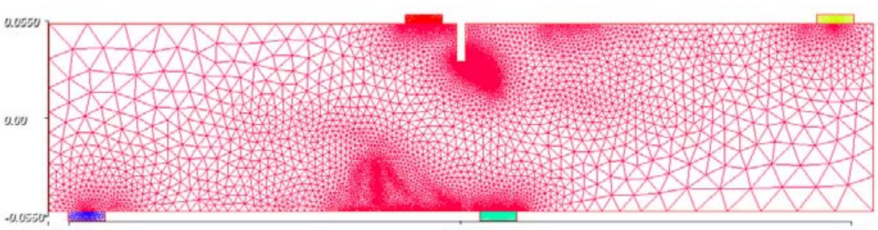


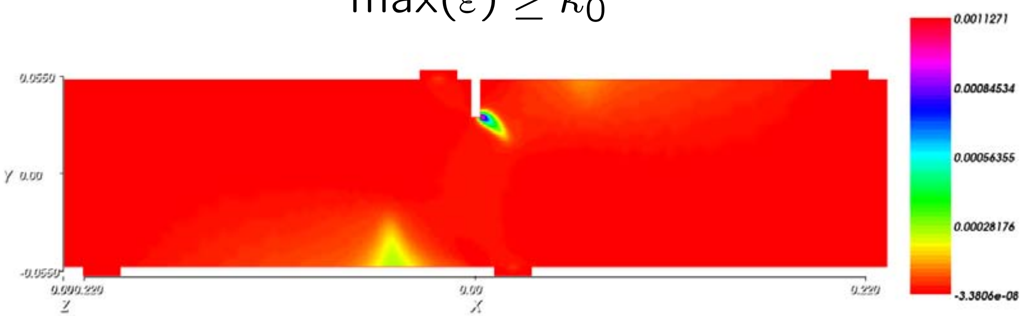
LKM, DOF: 29540

Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05



$\max(\bar{\varepsilon}) \geq \kappa_0$



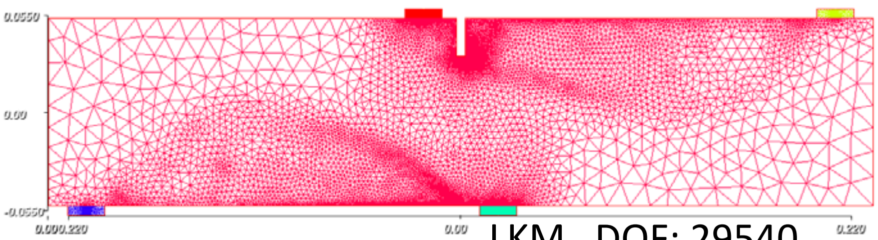


DOF: 29360

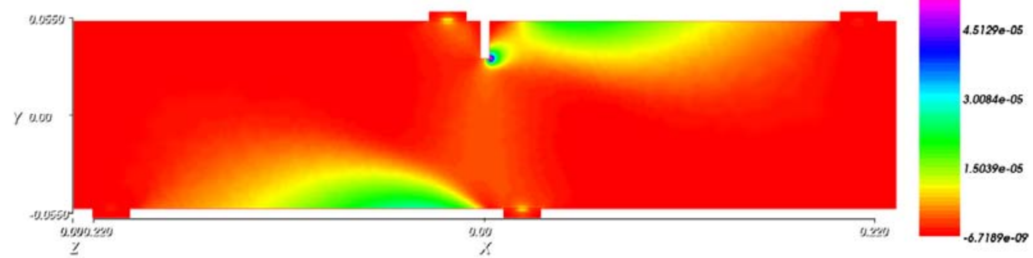
Ebar at [1,i+1]=[10,13]

Error-controlled adaptive simulations

$\bar{\varepsilon} < \kappa_0$



LKM, DOF: 29540

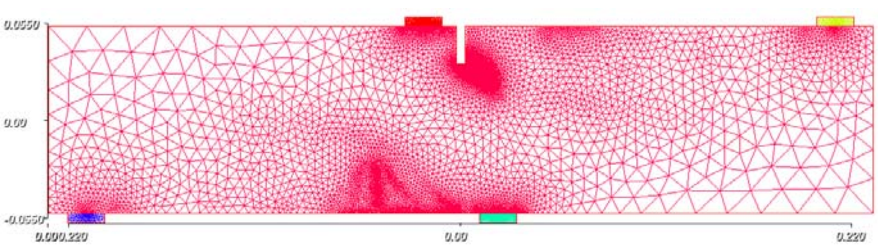


Ebar, Non-local Equiv. Strain on the LKM right BEFORE damage initiation; max(Ebar) = 6.01742e-05

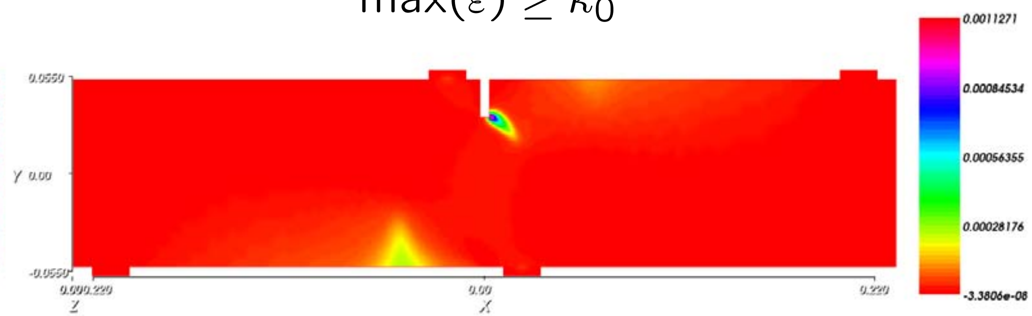
accordingly adapted mesh

evolution

$\max(\bar{\varepsilon}) \geq \kappa_0$



DOF: 29360

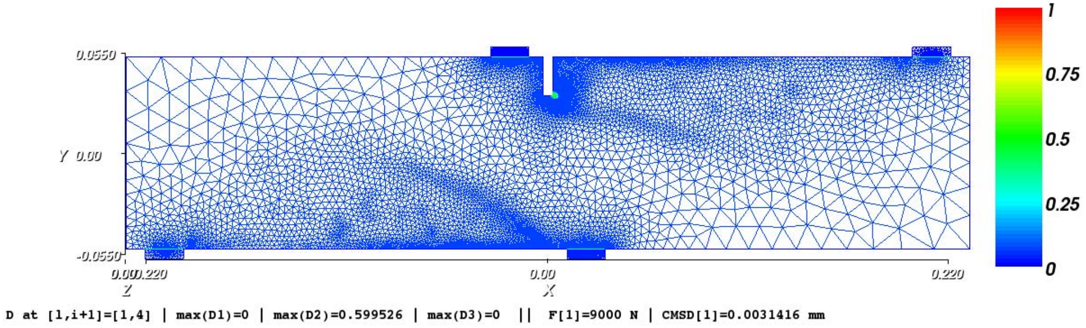


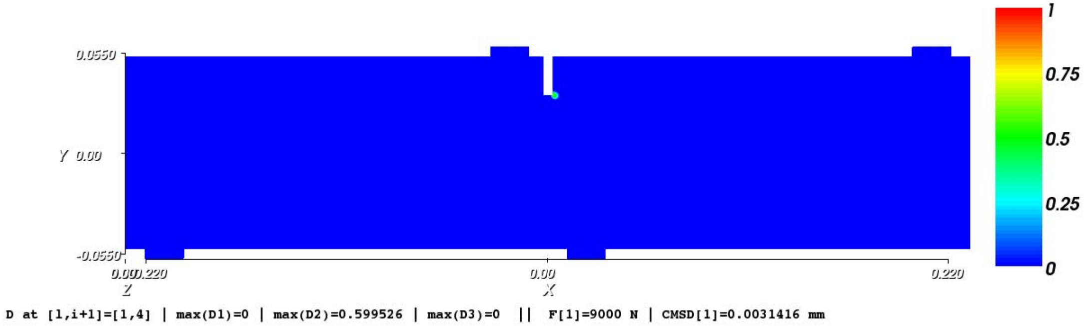
Ebar at [1,i+1]=[10,13]

Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 1

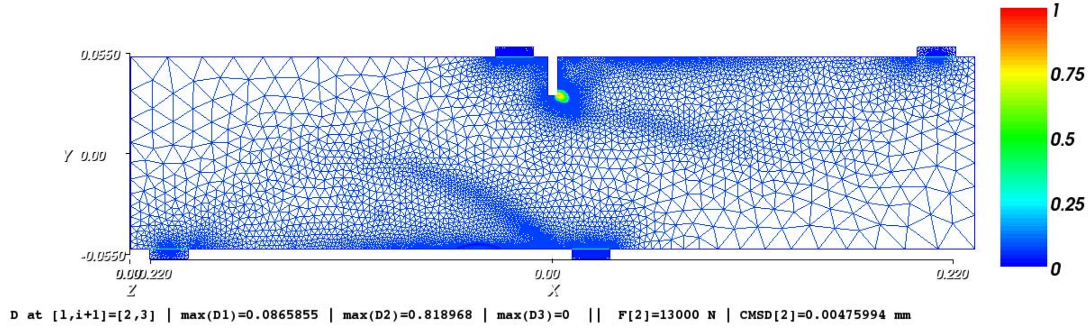


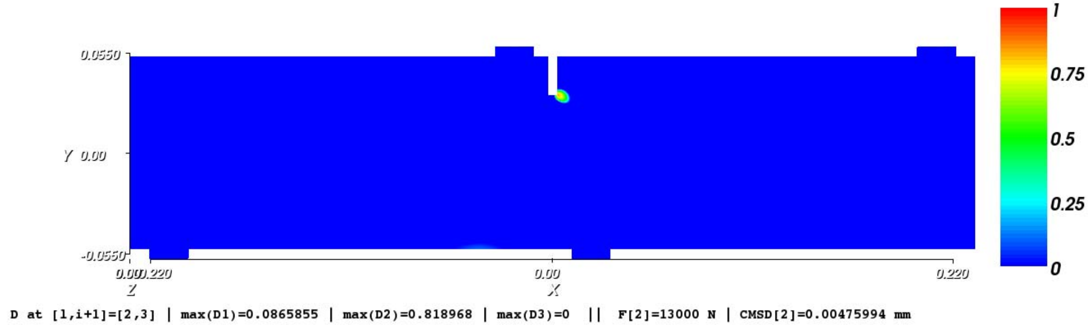


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 2

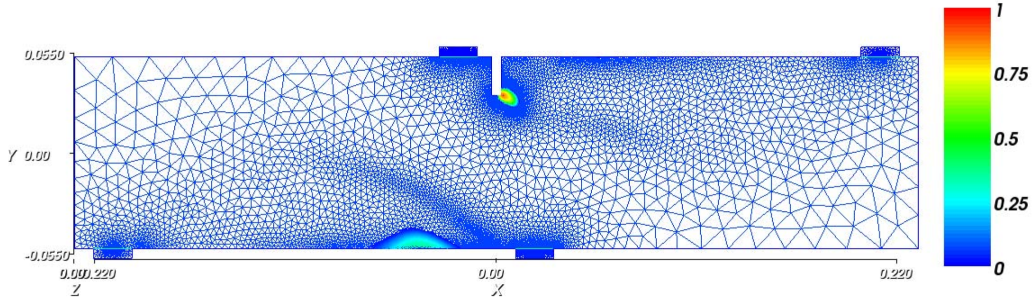




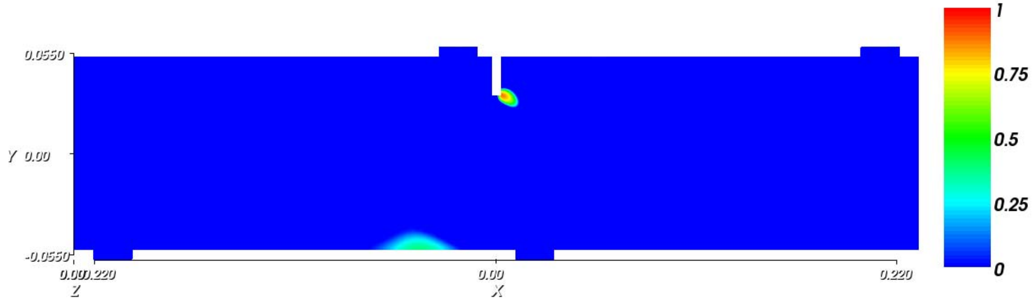
Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 3



D at [1,i+1]=[3,4] | max(D1)=0.375951 | max(D2)=0.884595 | max(D3)=0.0127604 || F[3]=15900 N | CMSD[3]=0.00608745 mm

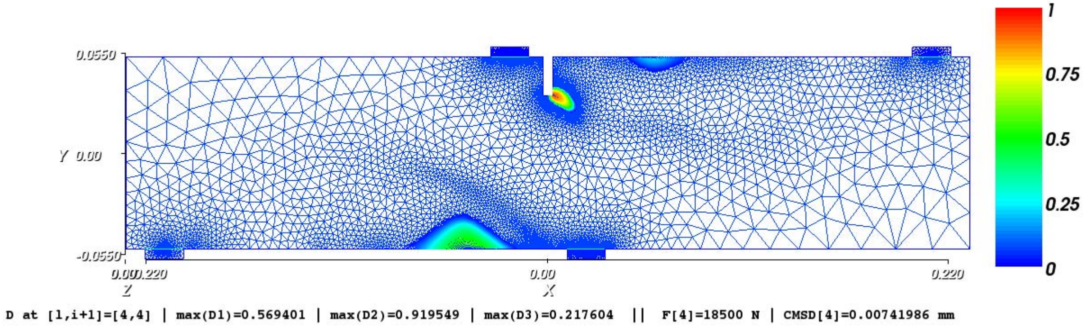


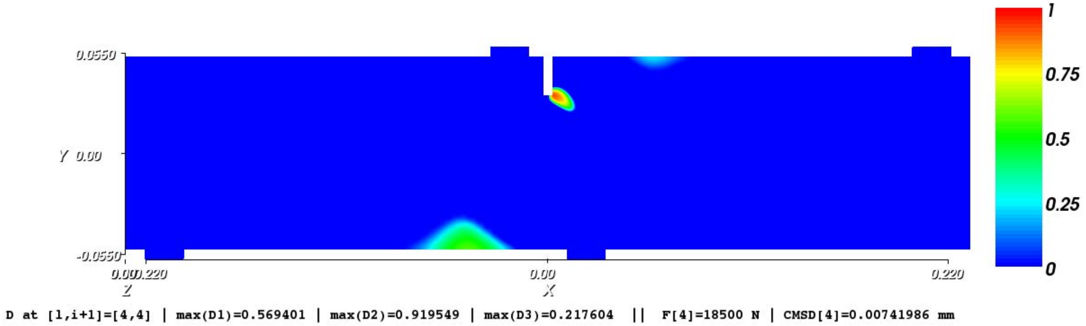
D at [1,i+1]=[3,4] | max(D1)=0.375951 | max(D2)=0.884595 | max(D3)=0.0127604 || F[3]=15900 N | CMSD[3]=0.00608745 mm

Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 4

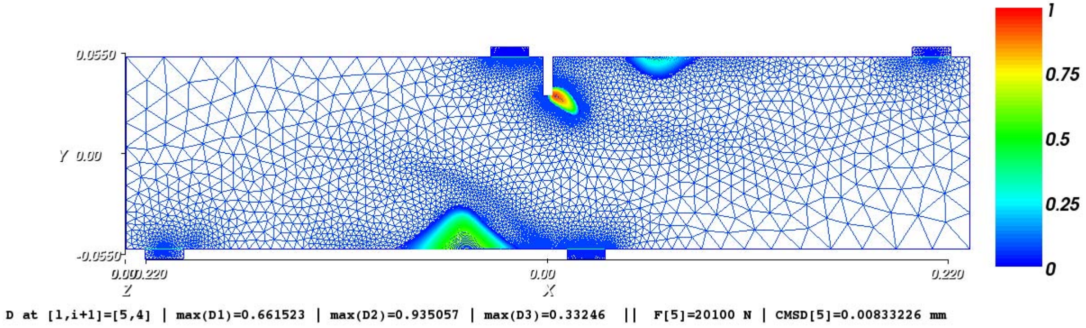


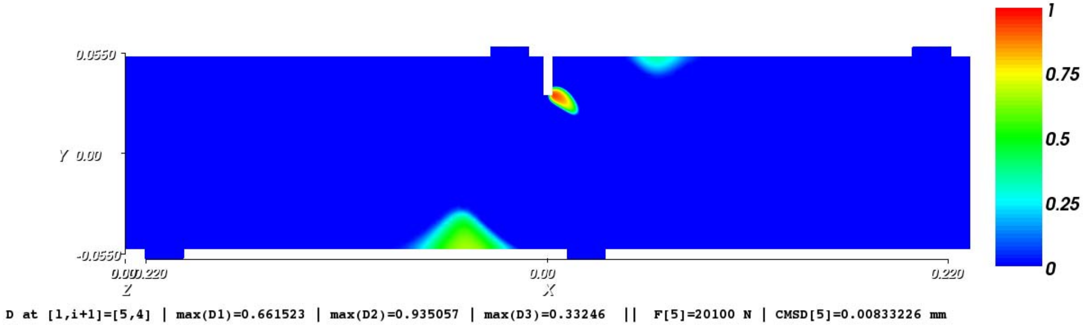


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 5

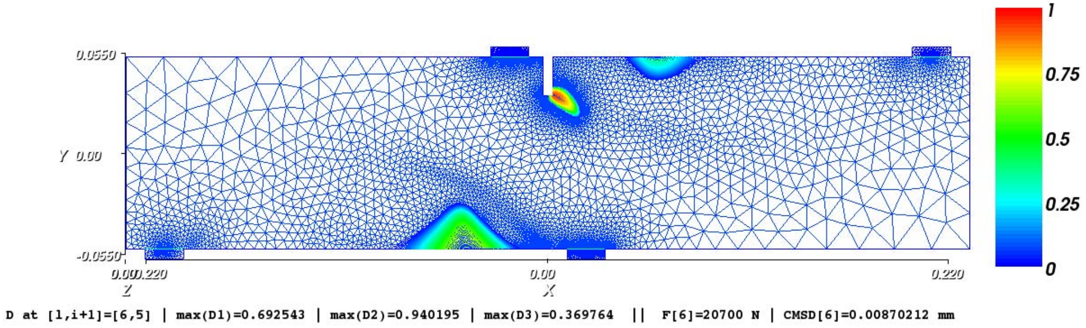


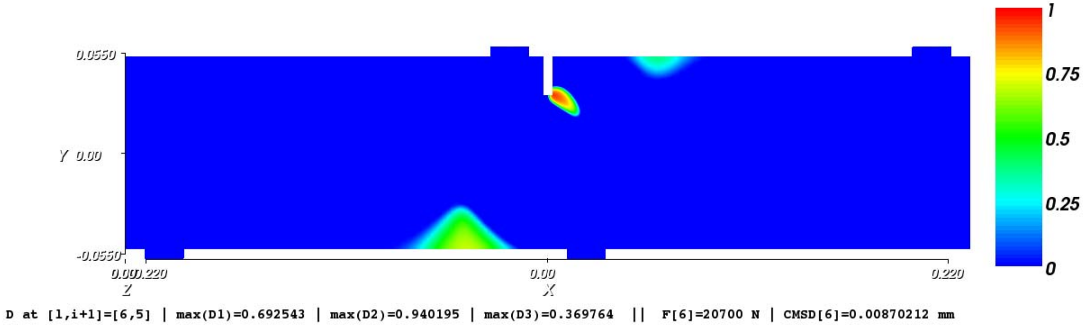


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 6

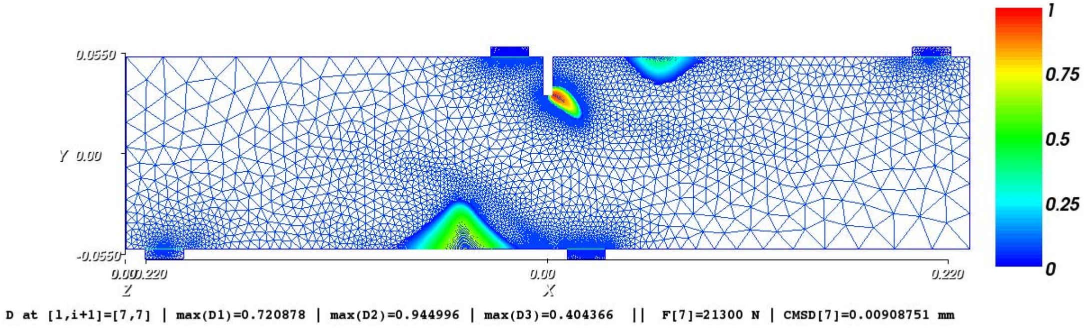


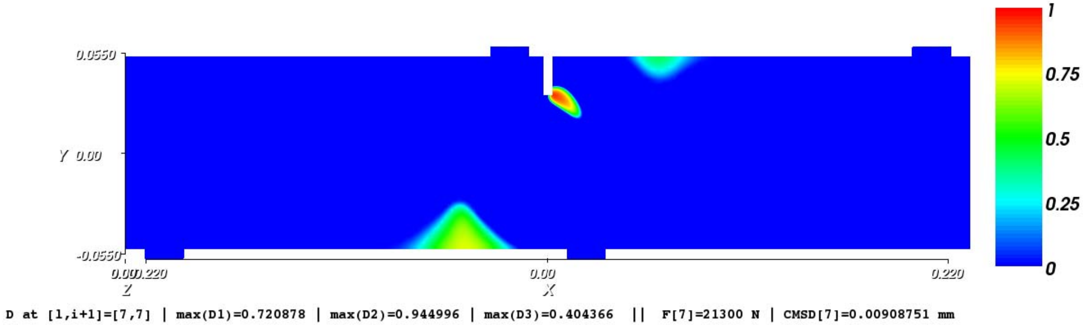


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 7

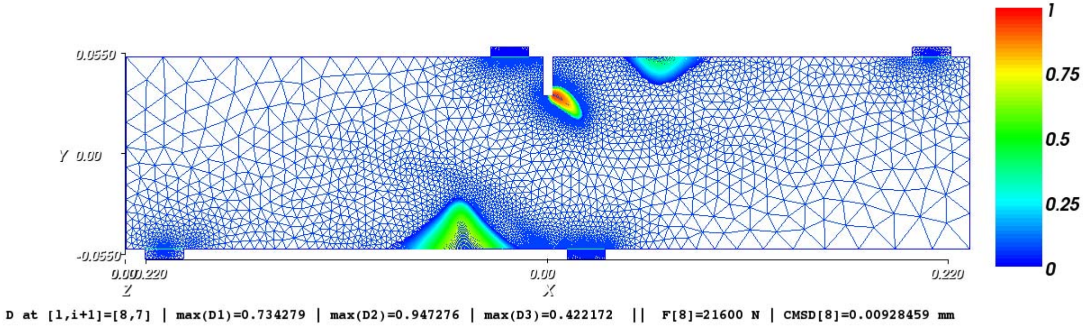


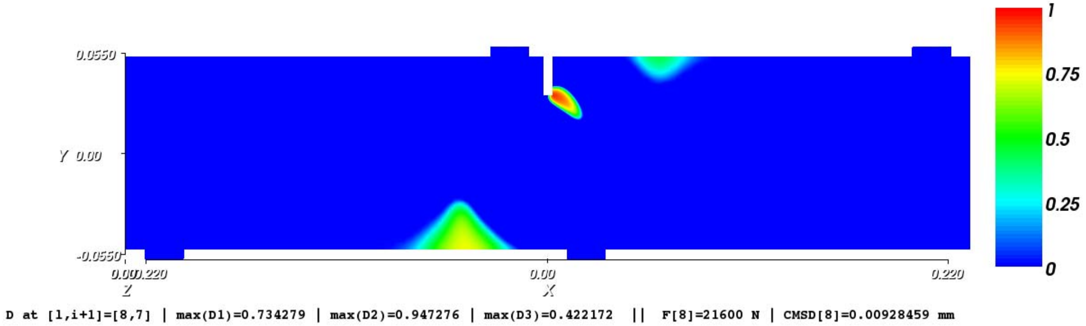


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 8

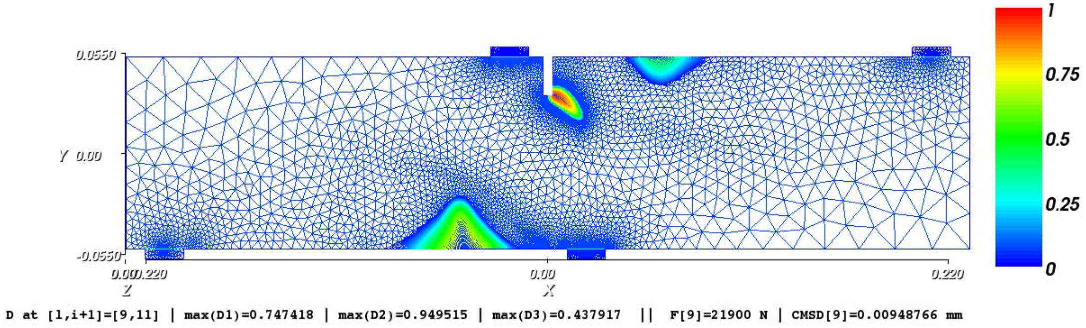


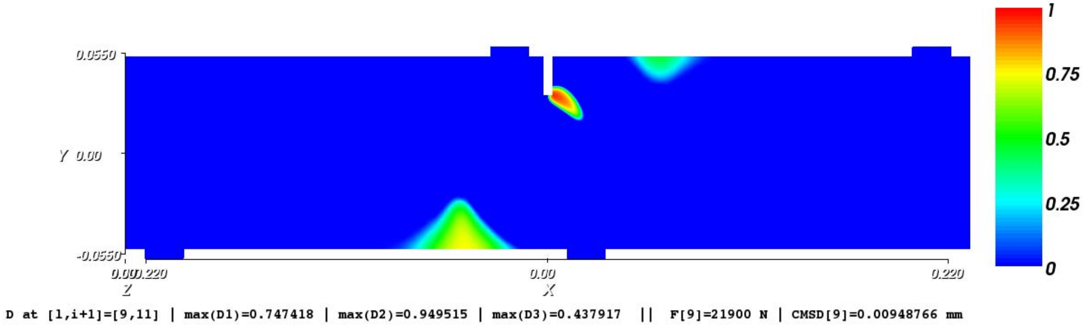


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

Loading Step 9

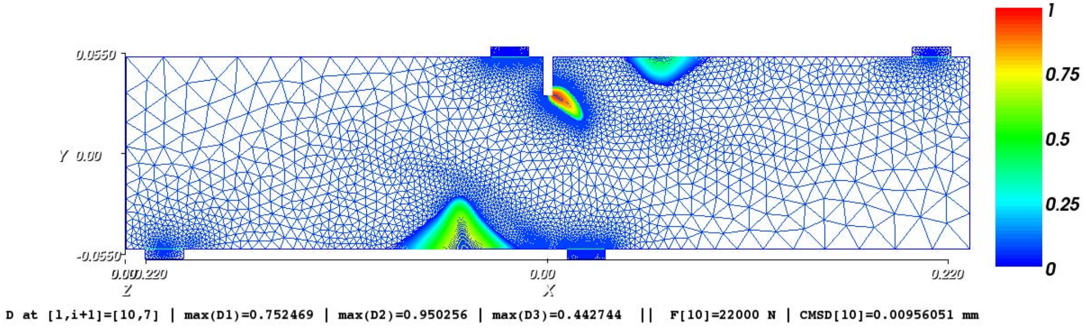


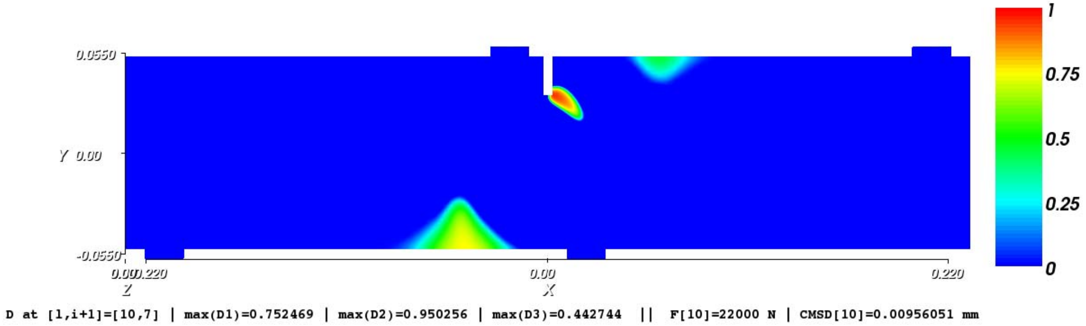


Error-controlled adaptive simulations

Damage evolution $D(\bar{\varepsilon})$ on a sequence of adaptive meshes

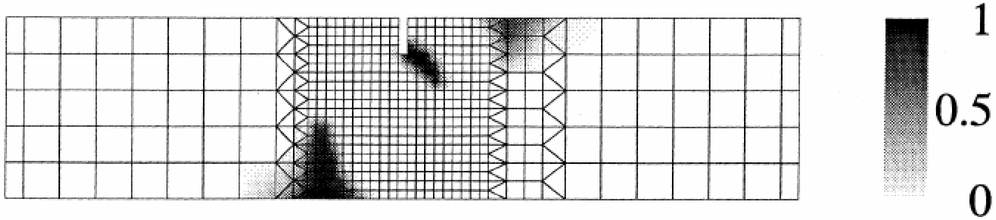
Loading Step 10 (**my peak load**)

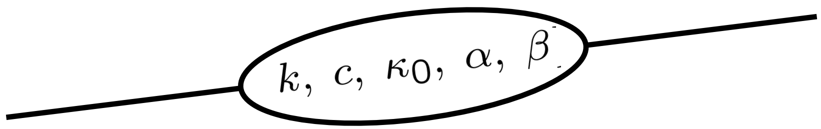


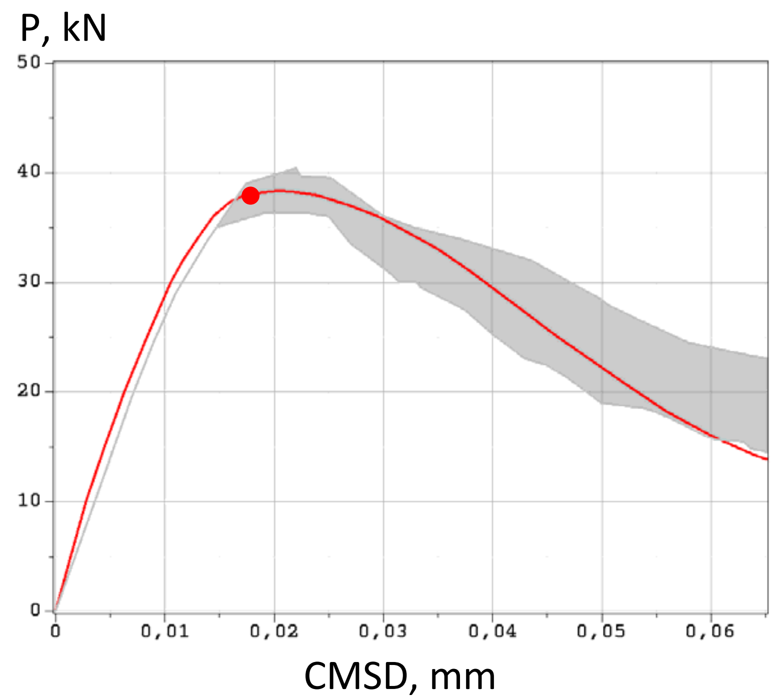


Error-controlled adaptive simulations

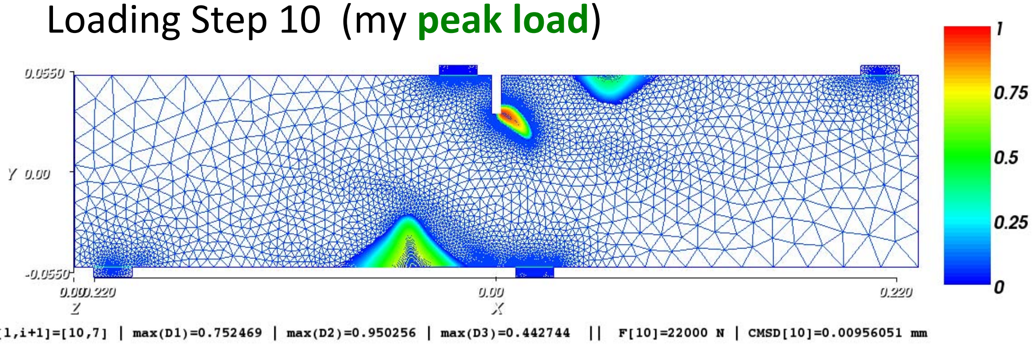
P=37 kN (right before the **peak load**)





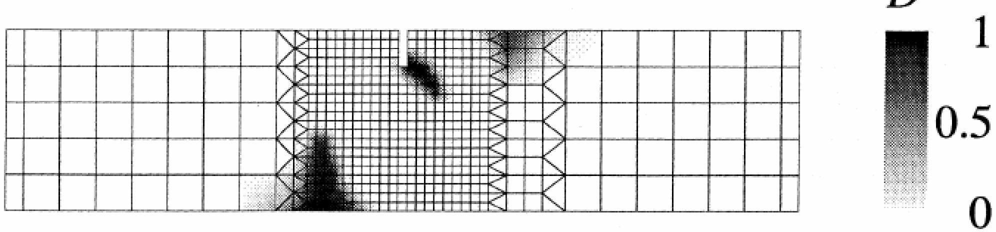


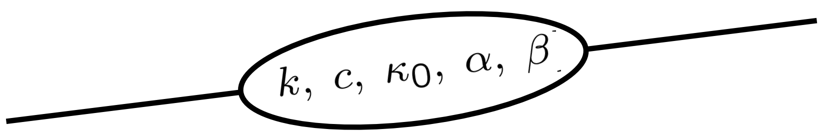
Loading Step 10 (my **peak load**)

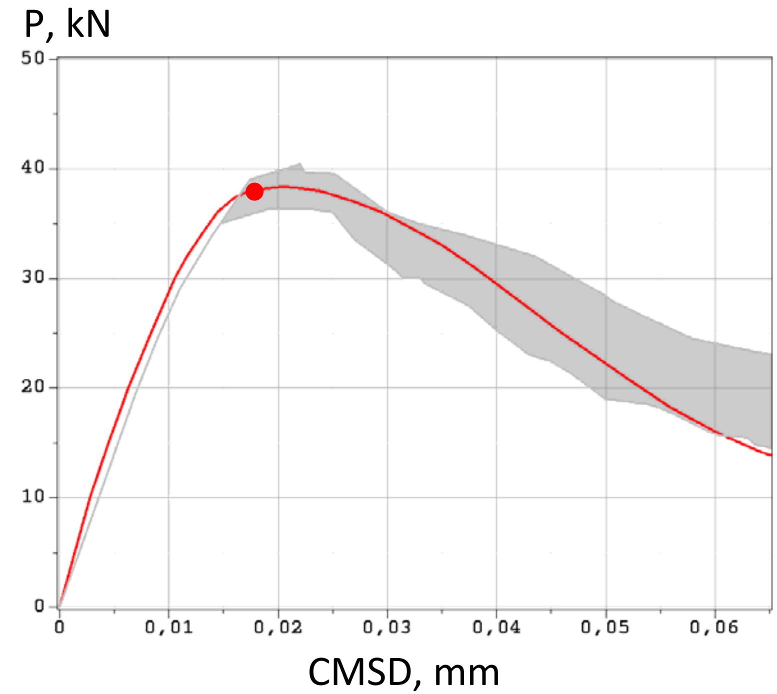


Error-controlled adaptive simulations

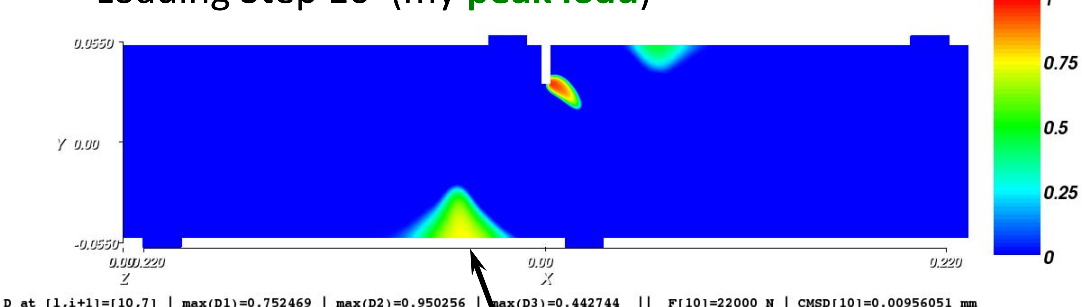
P=37 kN (right before the **peak load**)







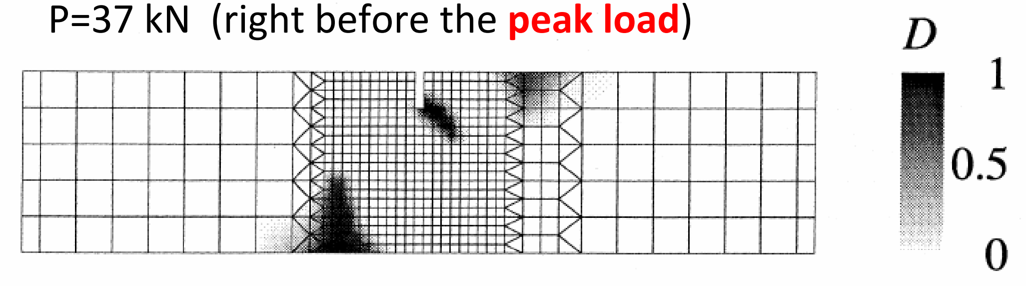
Loading Step 10 (my **peak load**)

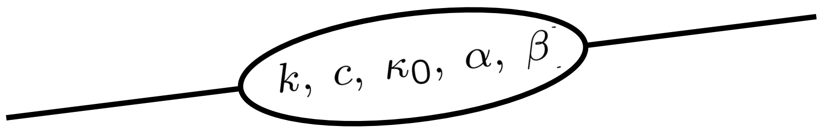


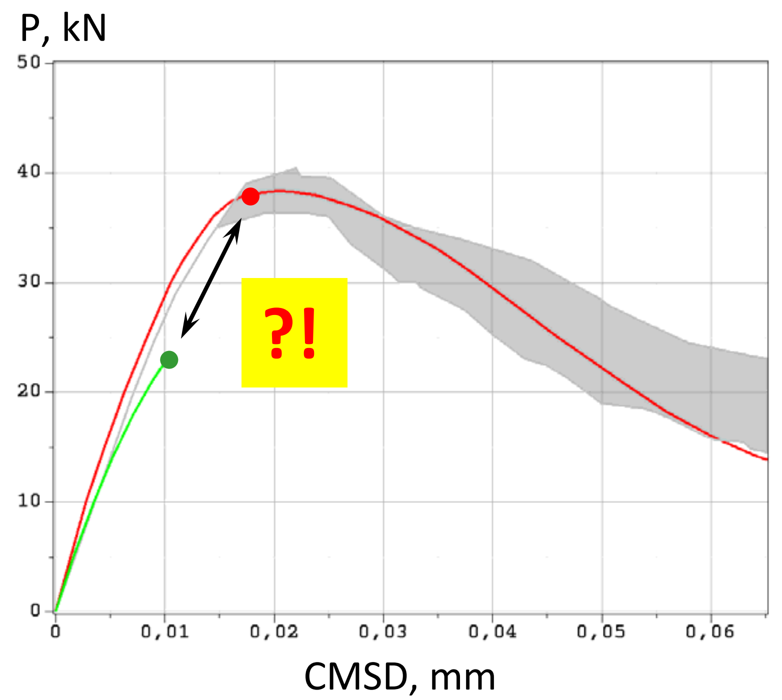
not that “severe”

Error-controlled adaptive simulations

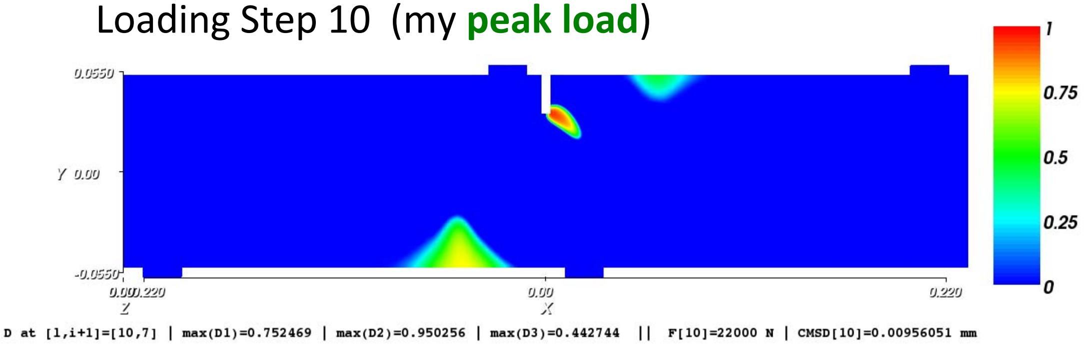
P=37 kN (right before the **peak load**)





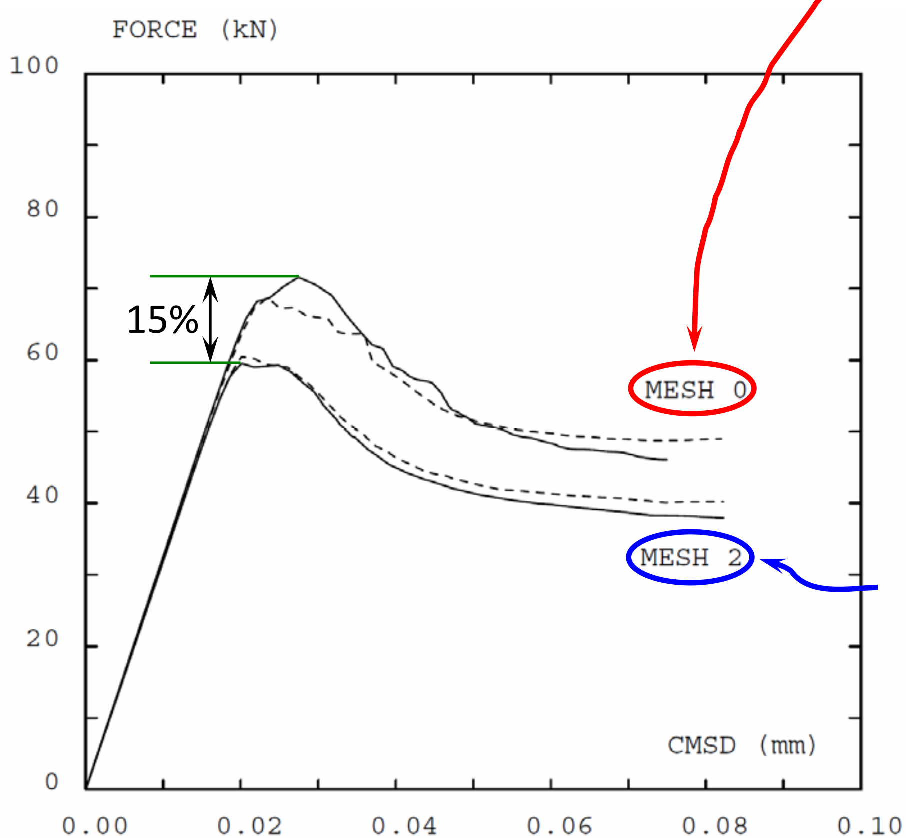


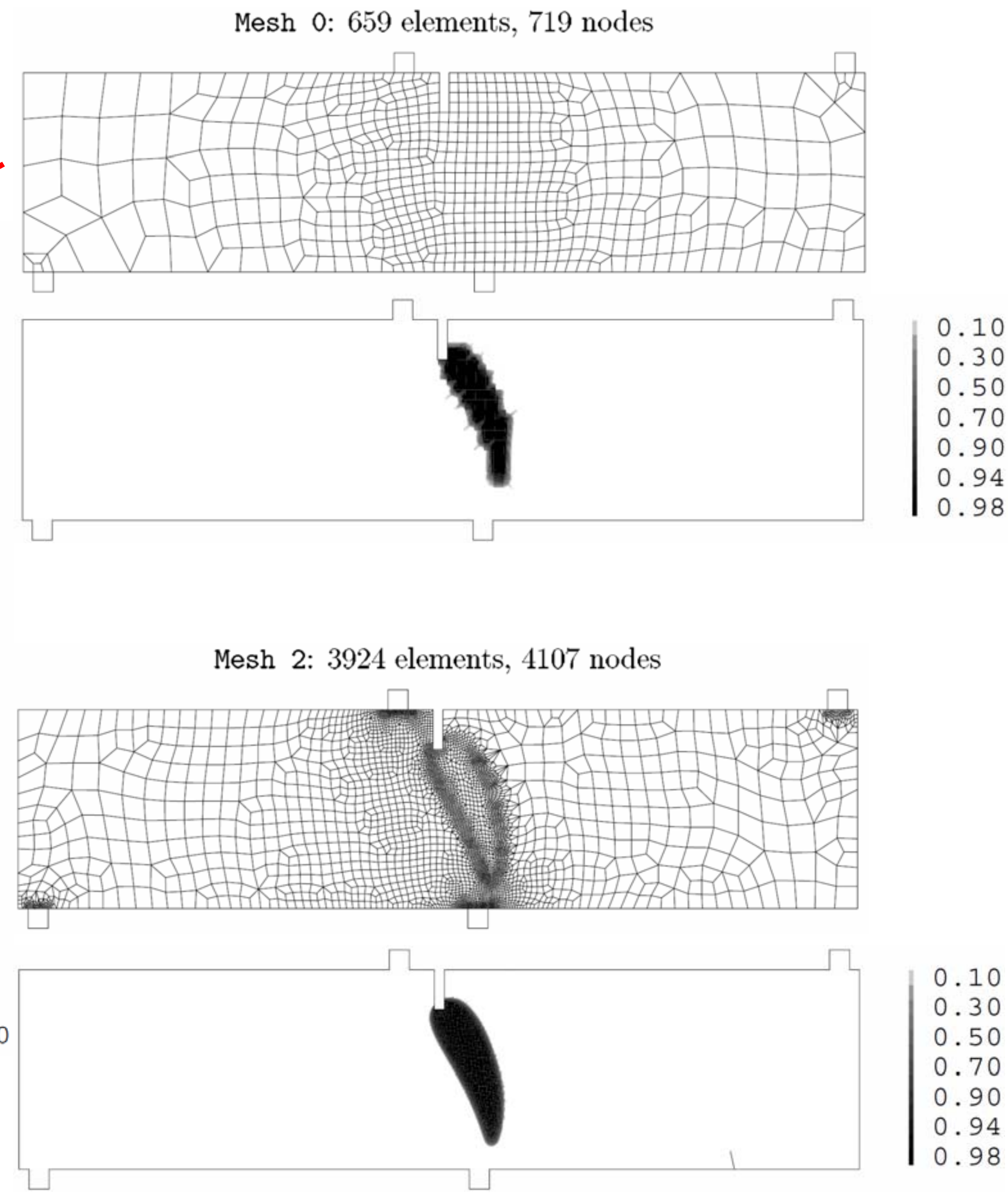
Loading Step 10 (my **peak load**)



"Mesh sensitivity" (not in a context of a **local** model formulation)

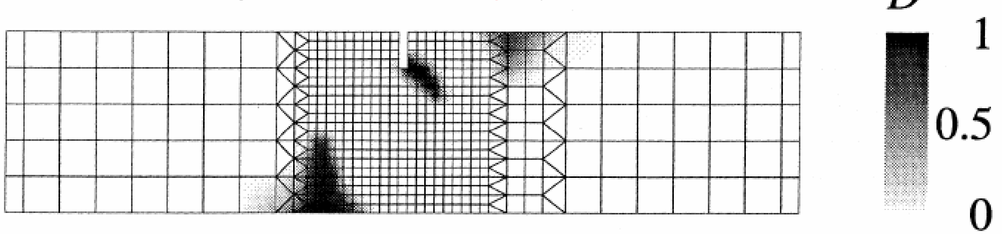
Rodriguez-Ferran, Huerta (2000)

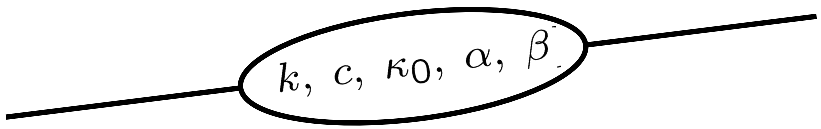


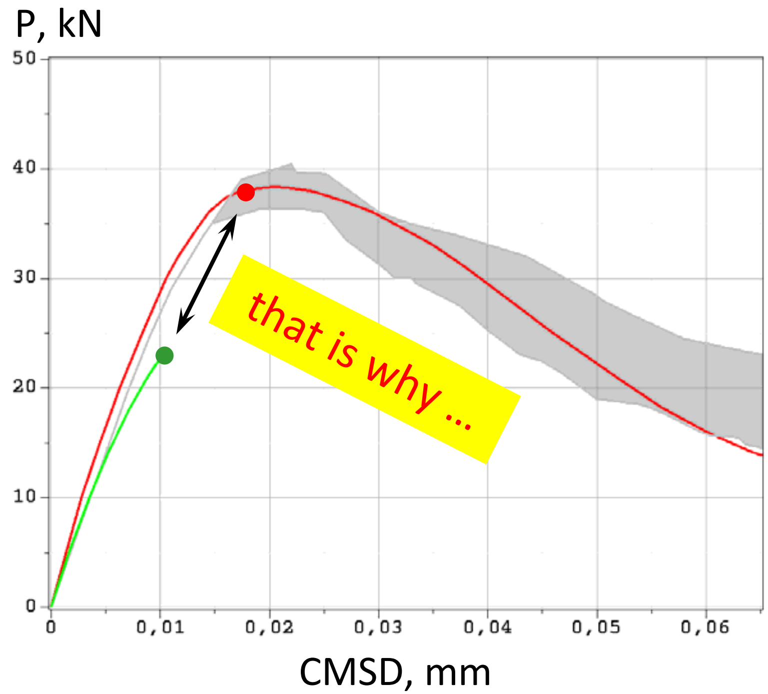


"Mesh sensitivity" (not in a context of a **local** model formulation)

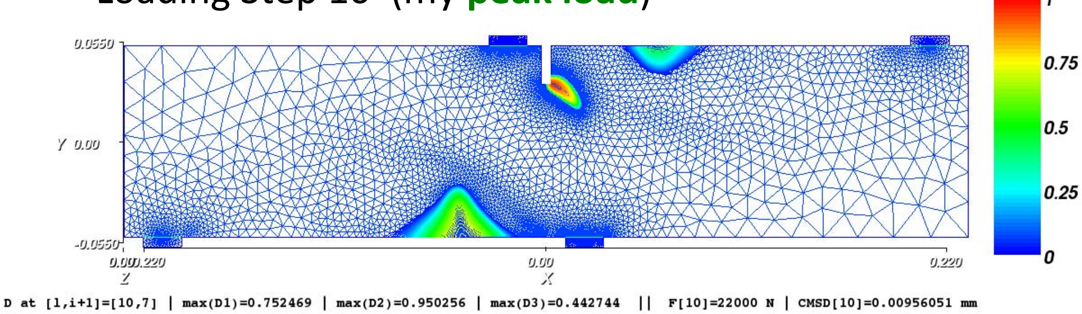
P=37 kN (right before the **peak load**)





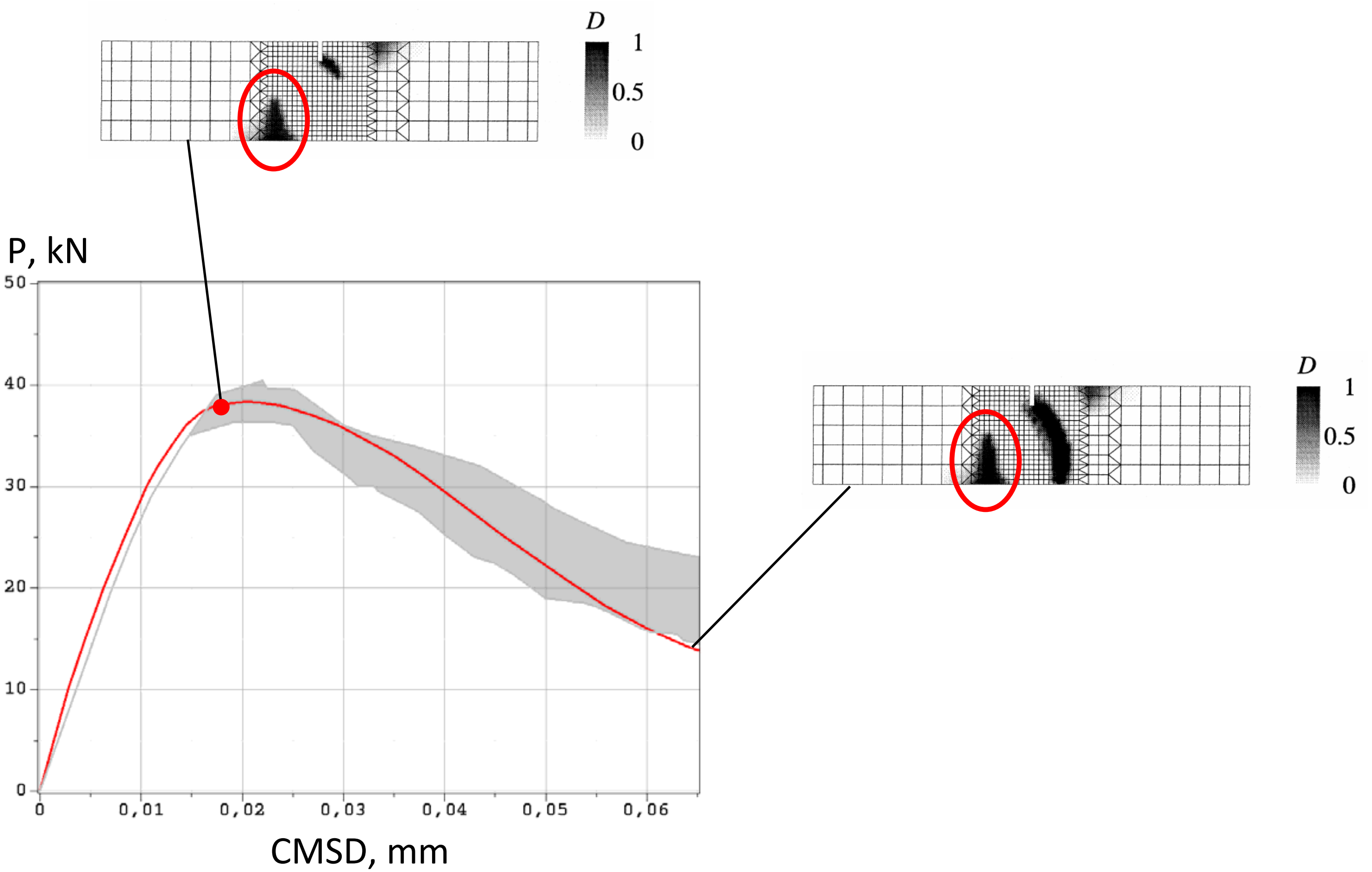


Loading Step 10 (my **peak load**)



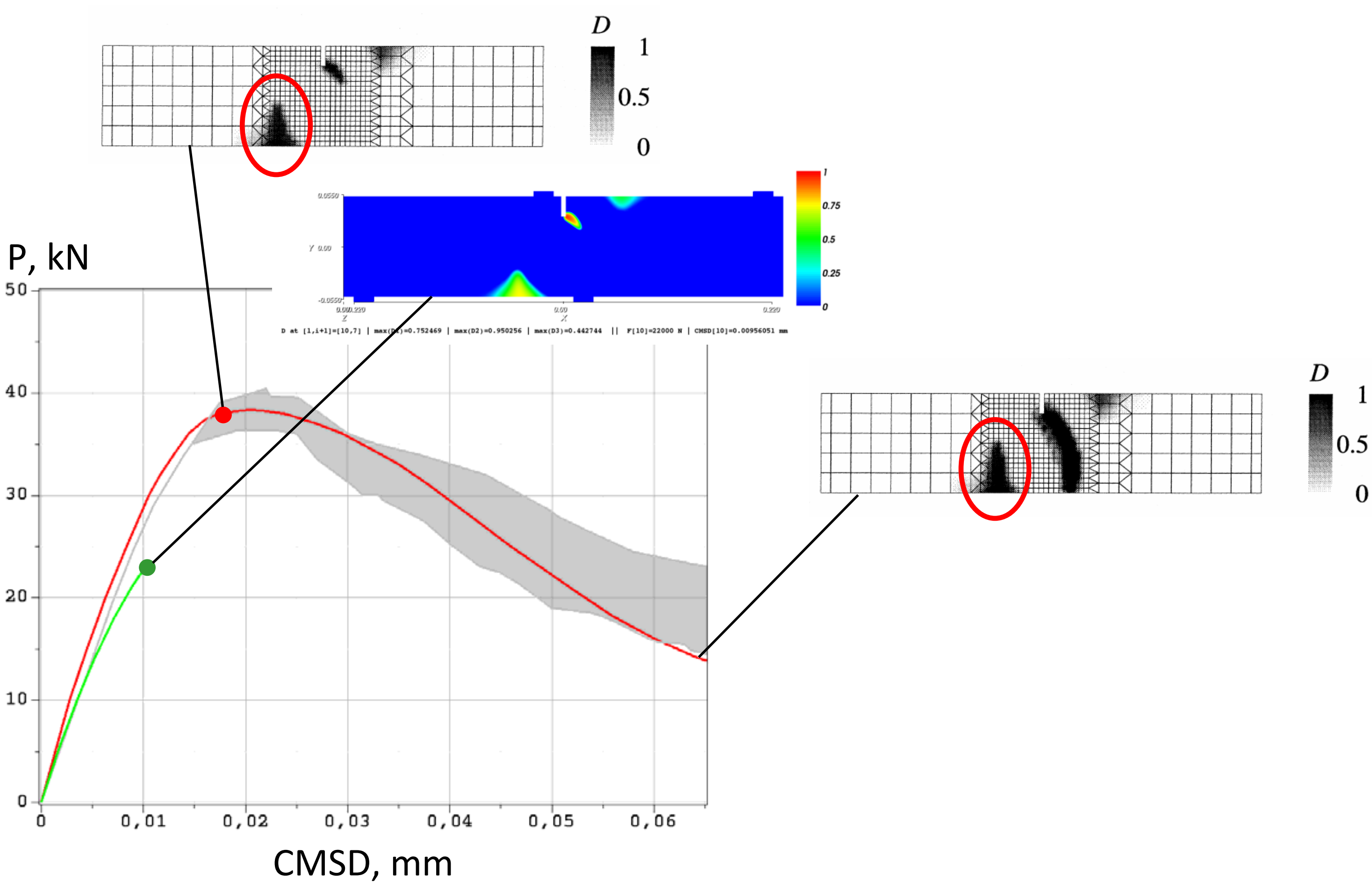
Pre-Conclusions

- modeling the complete failure with the damage approach only is not adequate – *spurious damage zones* (do they stand for cracks?)



Pre-Conclusions

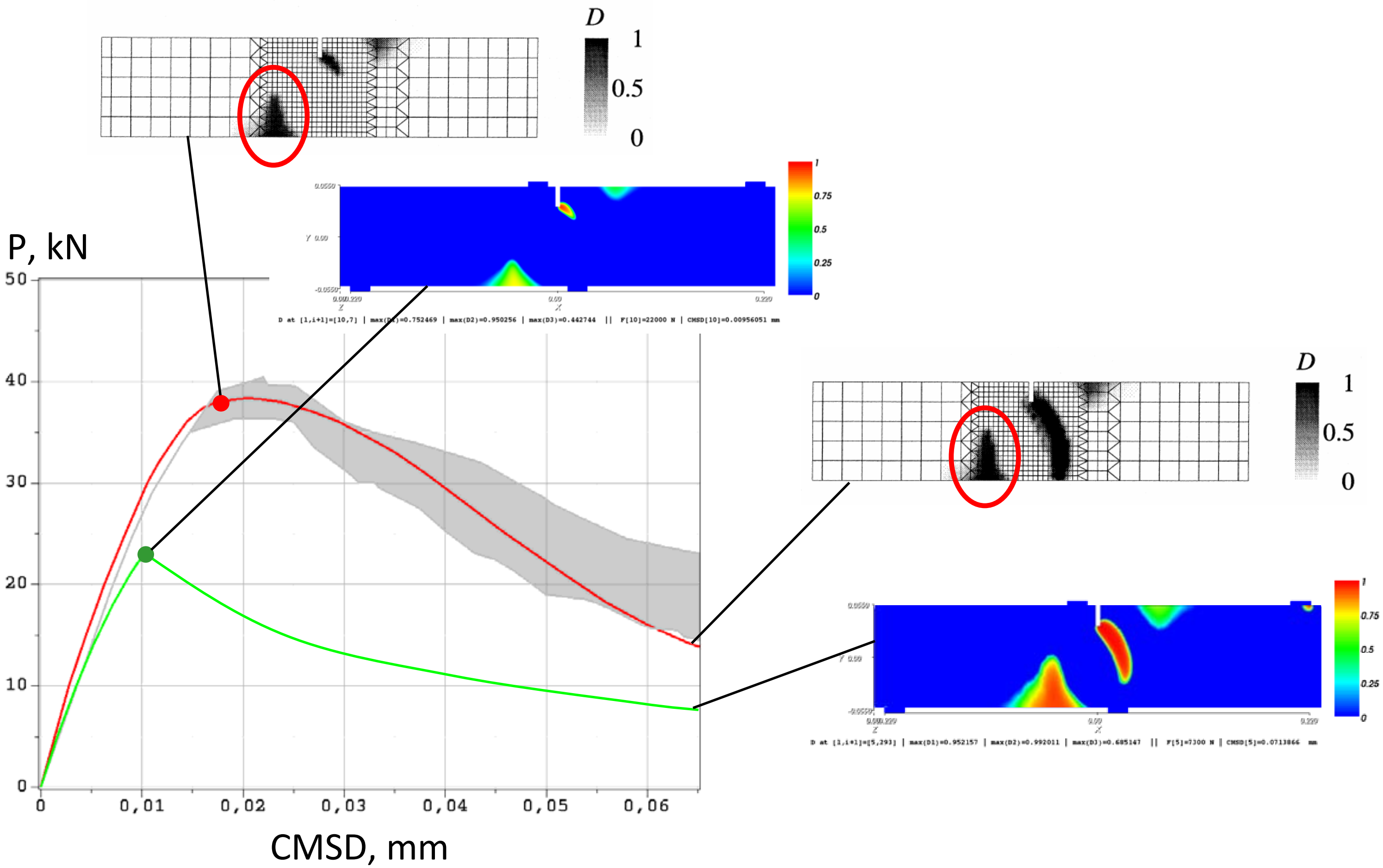
- modeling the complete failure with the damage approach only is not adequate – *spurious damage zones* (do they stand for cracks?)
- damage model parameters, calibrated on fixed (coarse/fine, non-adaptive) meshes – *non-optimal set of parameters*



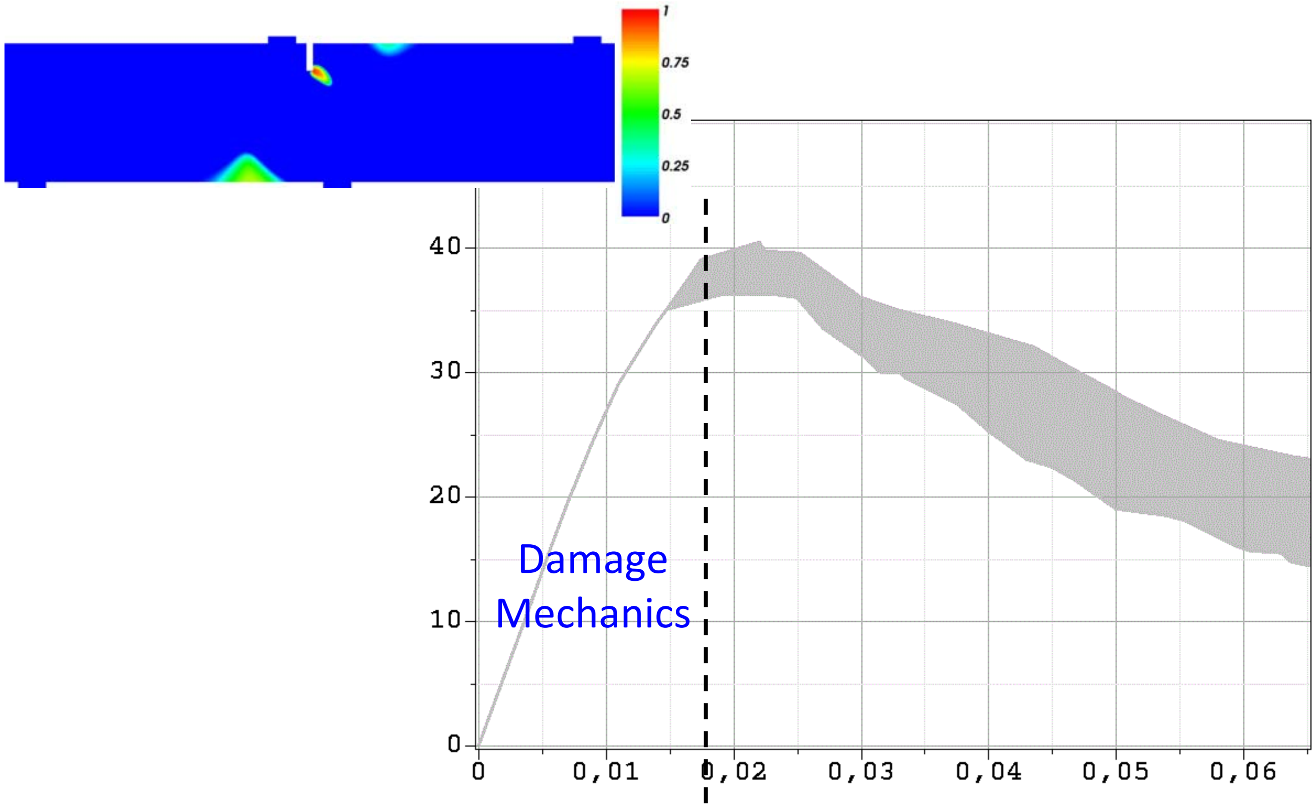
Pre-Conclusions

- modeling the complete failure with the damage approach only is not adequate – *spurious damage zones* (do they stand for cracks?)

- damage model parameters, calibrated on fixed (coarse/fine, non-adaptive) meshes – *non-optimal set of parameters*

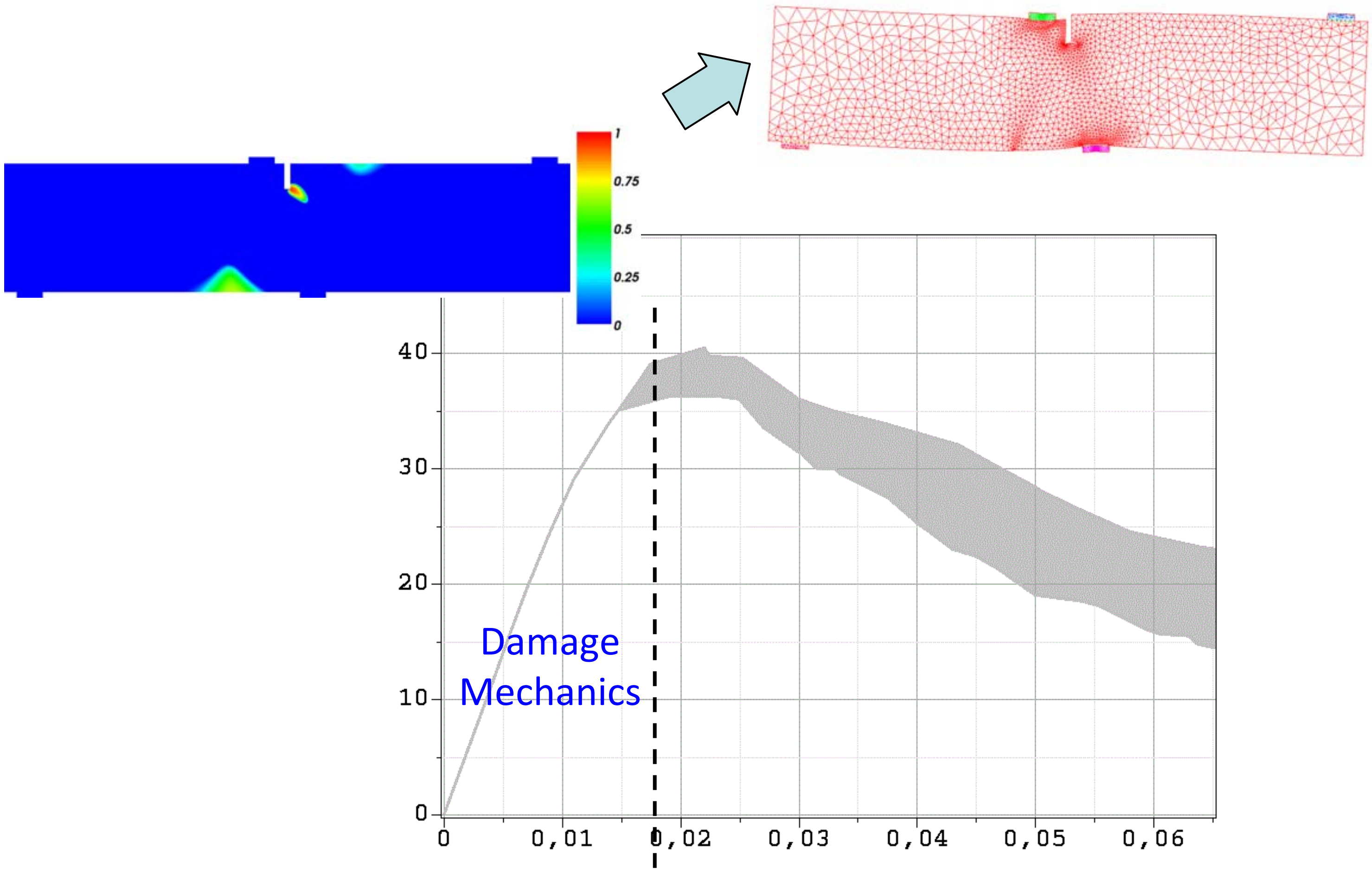


Motivation and goal



Motivation and goal

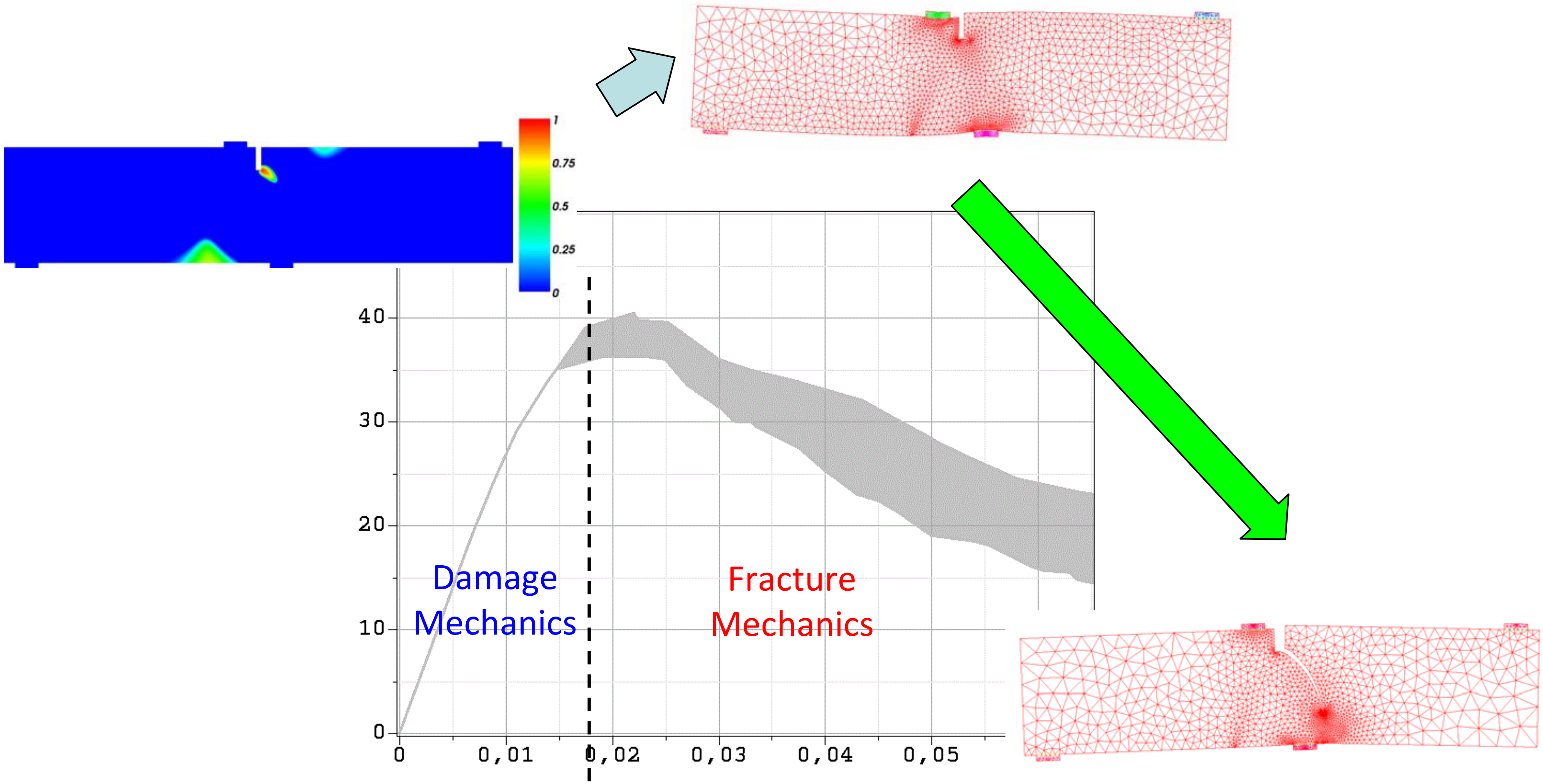
- transition from continuum damage to fracture
  - peak load vicinity
  - damage zones – crack nucleation
  - energetic equivalence★



★Mazars, Pijaudier-Cabot (1996):
From Damage to Fracture Mechanics and Conversely: a Combined Approach

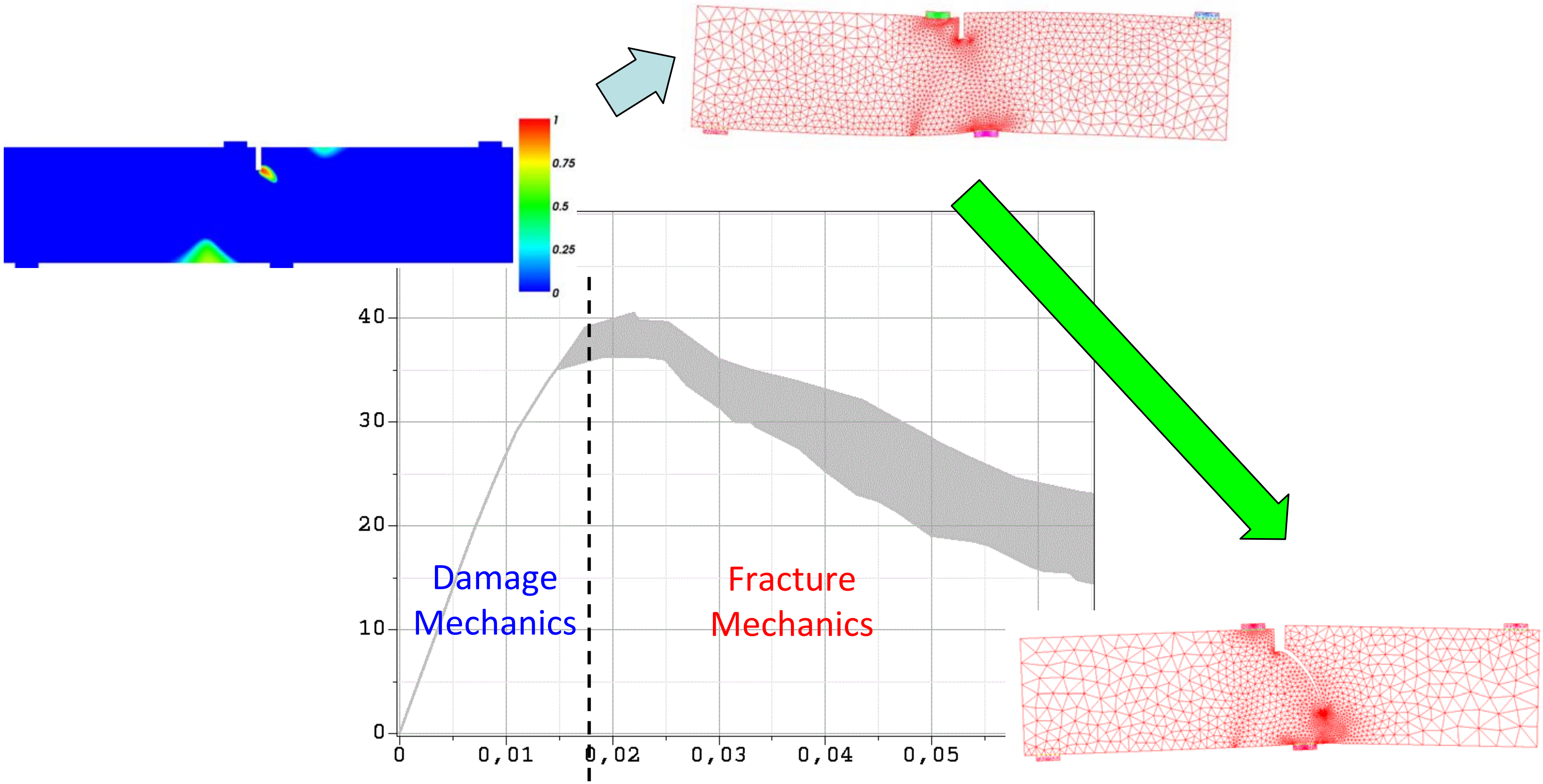
Motivation and goal

- transition from continuum damage to fracture



Motivation and goal

- transition from continuum damage to fracture



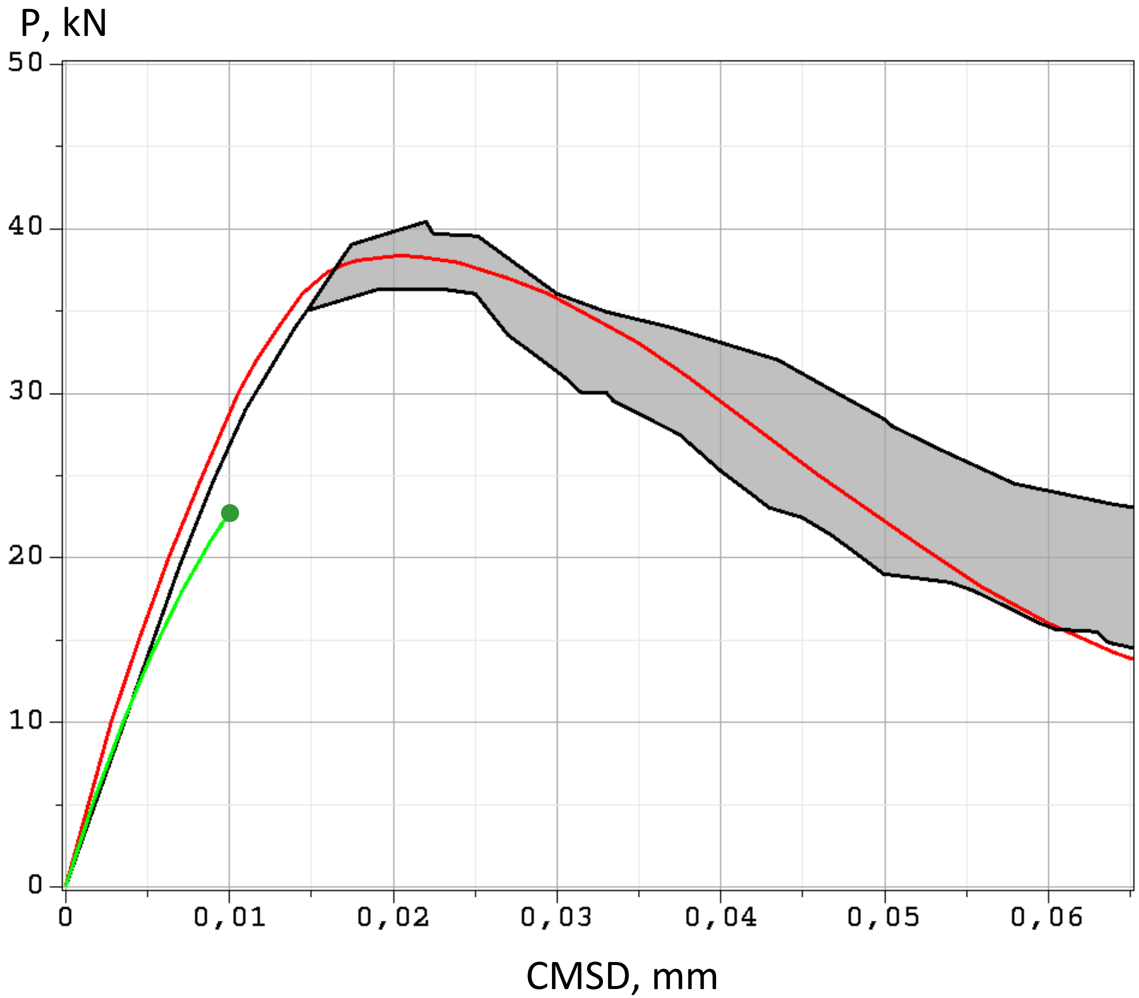
- lower order elements (P1-triangles)
- error-controlled Mesh Adaptivity for the entire simulation process

Error-controlled adaptive simulations

Parameters calibration

$\alpha := 0.96$
$\beta := 100$
$\kappa_0 := 6 \cdot 10^{-5}$

$k := 15$
$c := 1 \cdot 10^{-6}\ \text{m}^2$



Error-controlled adaptive simulations

Parameters calibration: quasi-optimal set

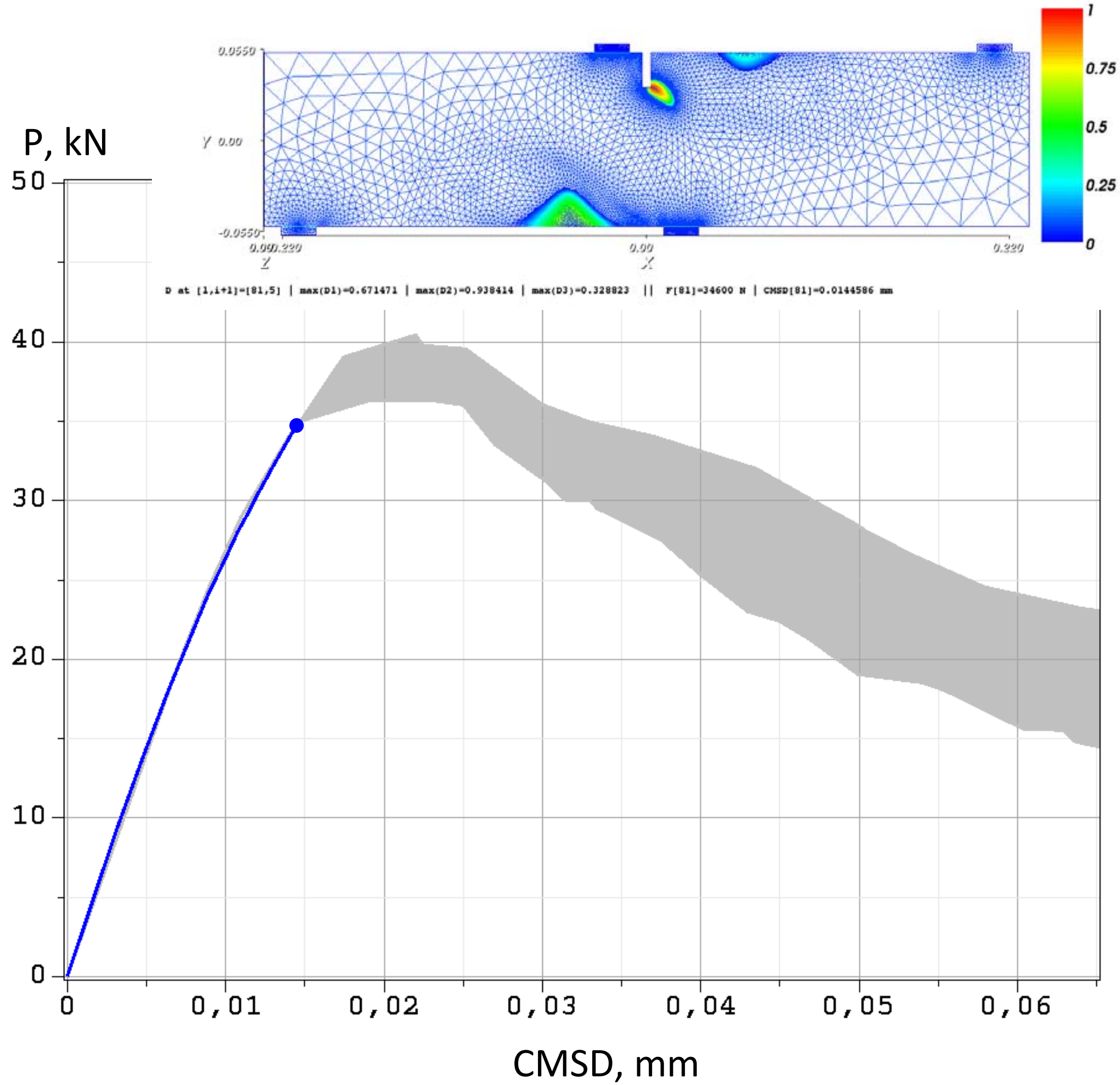
$\alpha := 0.96$

$\beta := 100$

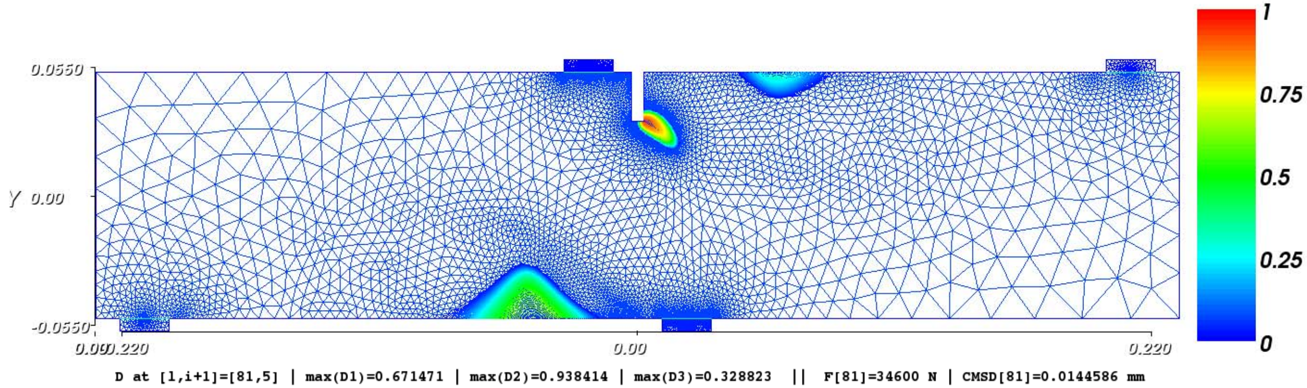
$\kappa_0 := 6 \cdot 10^{-5}$ → $1.1 \cdot 10^{-4}$

$k := 15$

$c := 1 \cdot 10^{-6}\ \mathrm{m}^2$ → $0.6 \cdot 10^{-6}$



Damage-to-fracture transition



Transformation of damage zones into equivalent cracks (Mazars, Pijaudier-Cabot, 1996):

$$A_e = \frac{\int_\Omega \int_0^{D(x)} -Y \; dD dx}{G_c}$$

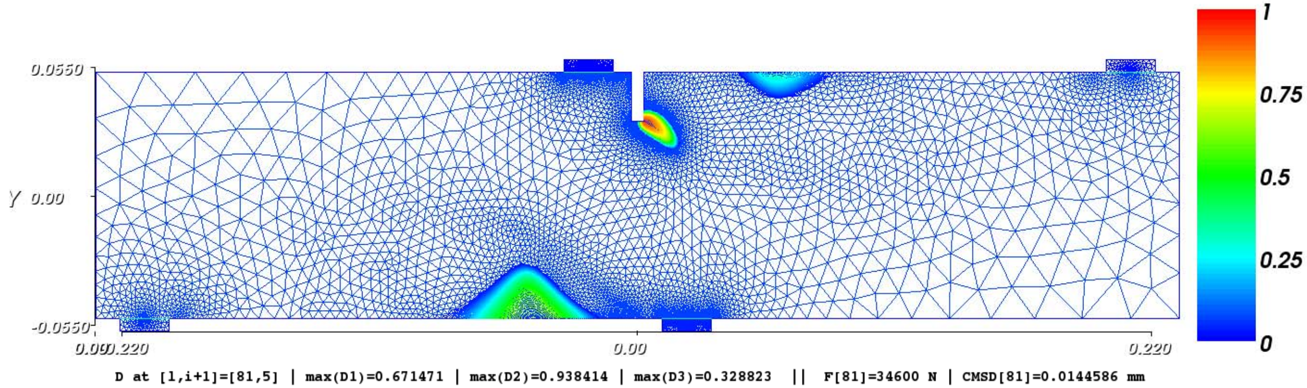
area of equivalent crack

damage energy release rate

$$Y := \frac{\partial \Psi}{\partial D} = -\tfrac{1}{2}\varepsilon : \mathbb{C} : \varepsilon$$

threshold value of the fracture energy release rate

Damage-to-fracture transition



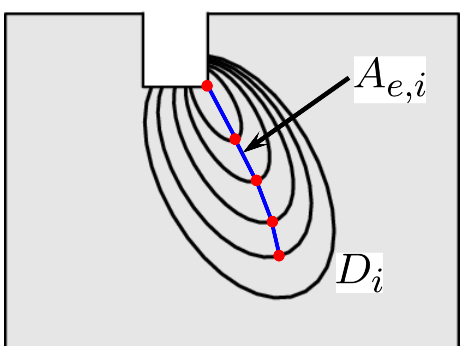
Transformation of damage zones into equivalent cracks (Mazars, Pijaudier-Cabot, 1996):

1

$$A_{e,i} = \frac{\int_{\Omega} -Y \; D_i(x) dx}{G_c} \qquad i = 1, 2, 3$$

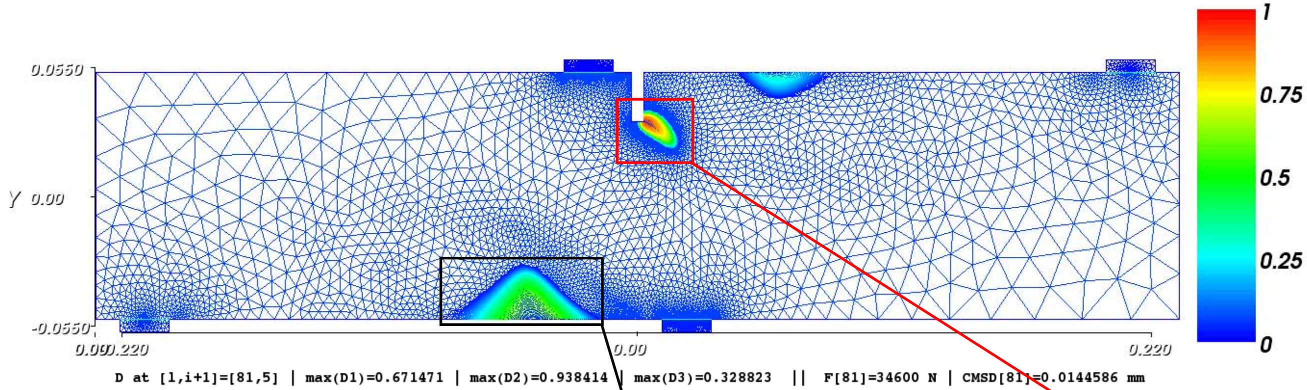
length of equivalent crack

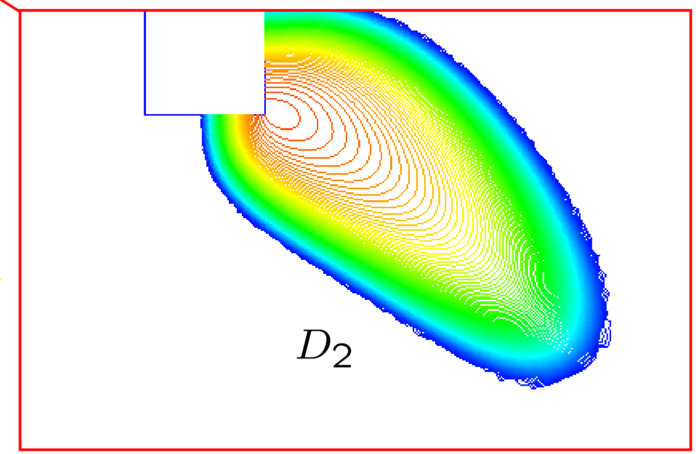
2



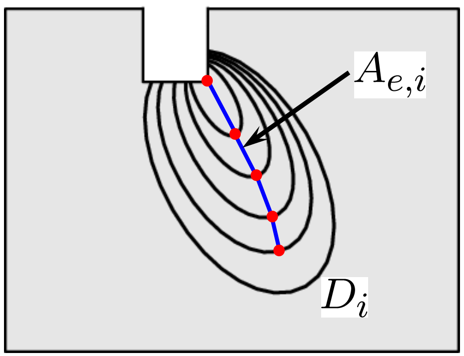
equivalent crack geometry, suggested by the damage profile

Damage-to-fracture transition

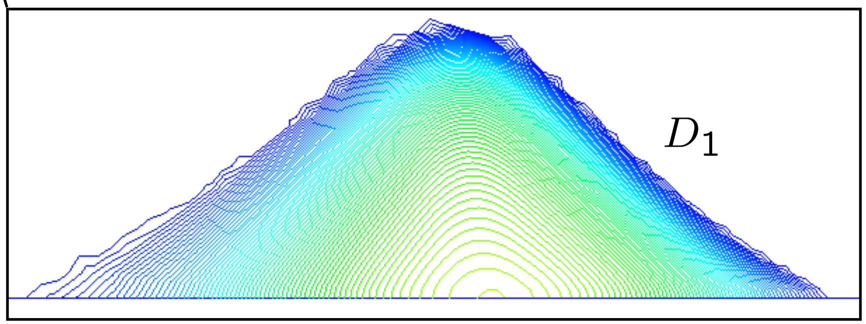




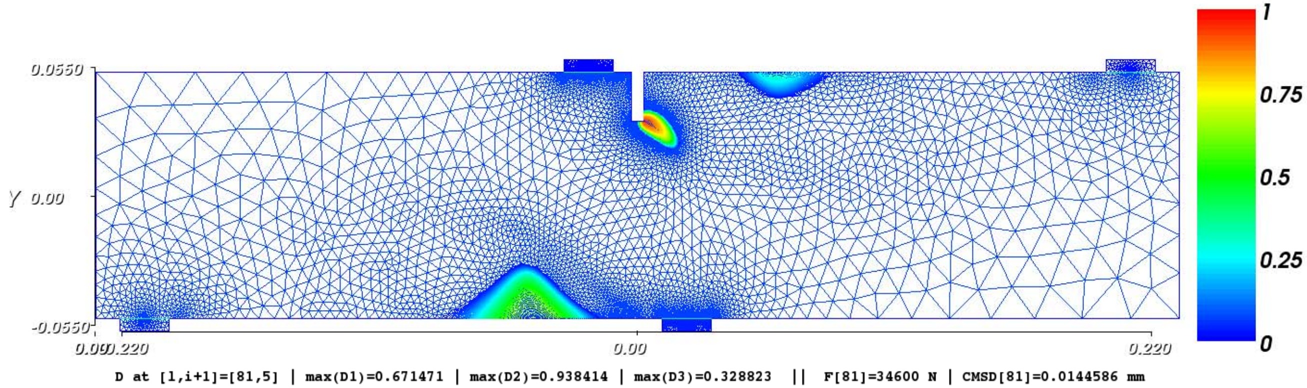
doable



equivalent crack geometry, suggested by the damage profile



Damage-to-fracture transition



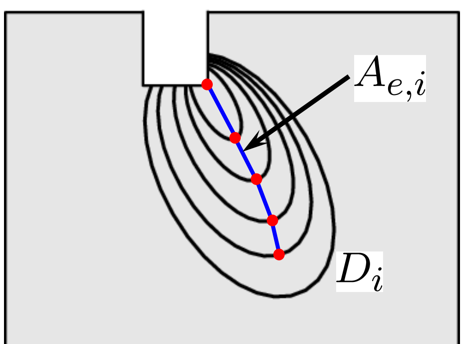
Transformation of damage zones into equivalent cracks (Mazars, Pijaudier-Cabot, 1996):

1

$$A_{e,i} = \frac{\int_{\Omega} -Y \; D_i(x)dx}{G_c} \qquad i = 1, 2, 3$$

length of equivalent crack

2



equivalent crack geometry, suggested by the damage profile

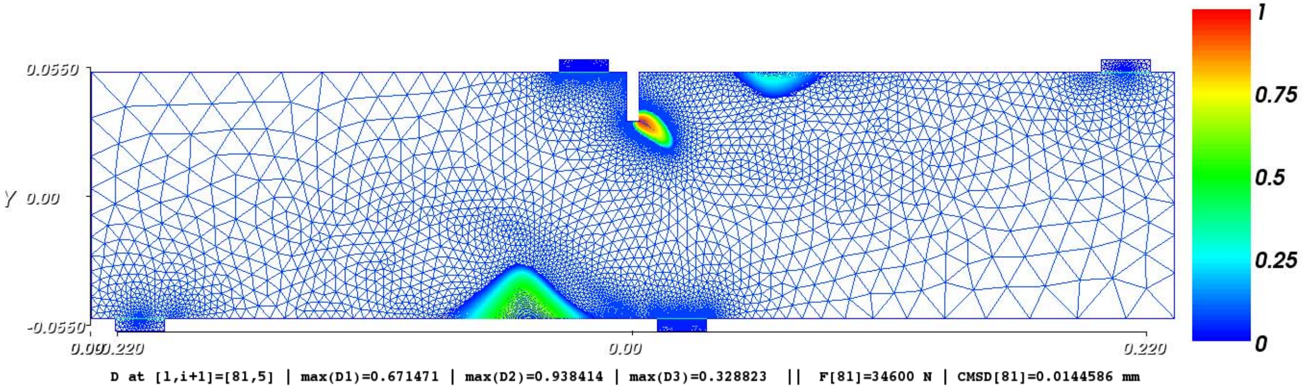
3

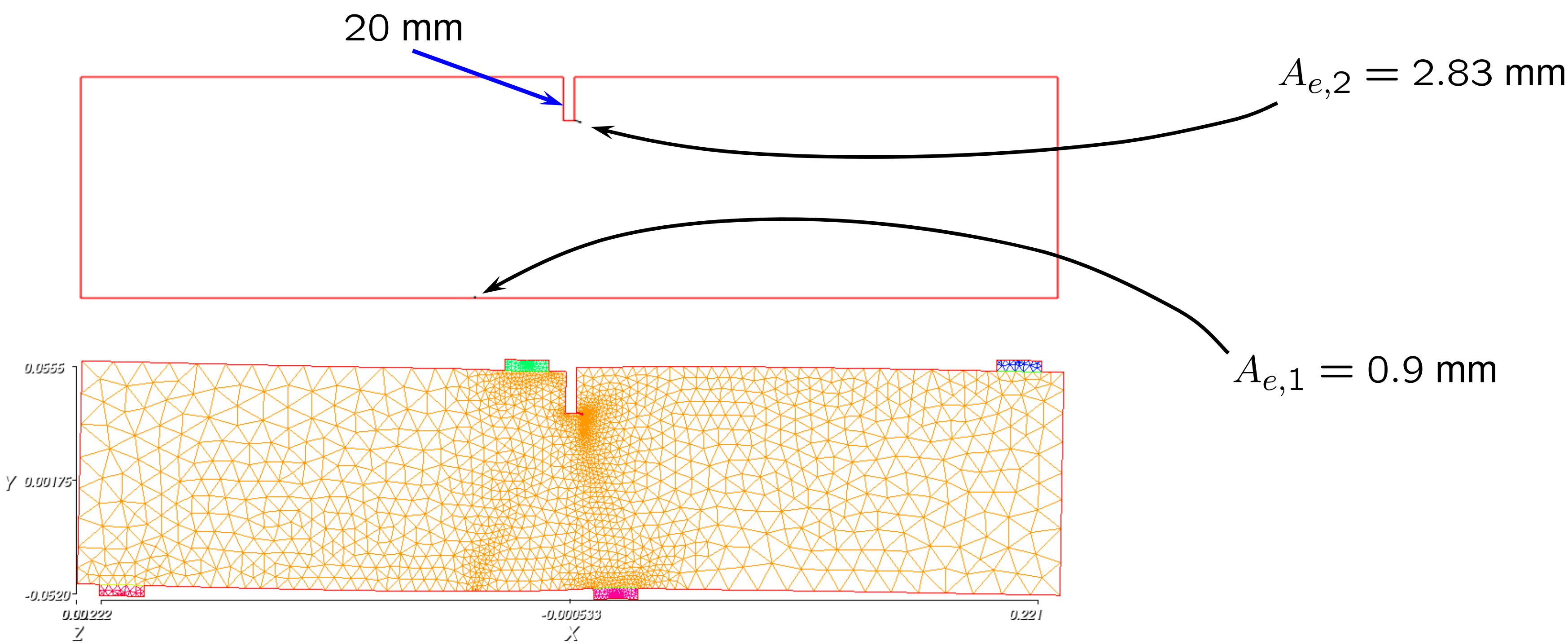
Schlangen (1993):

$G_c = 115.1 \pm 8.3$ N/m

$(90 \div 230$ N/m)

Damage-to-fracture transition

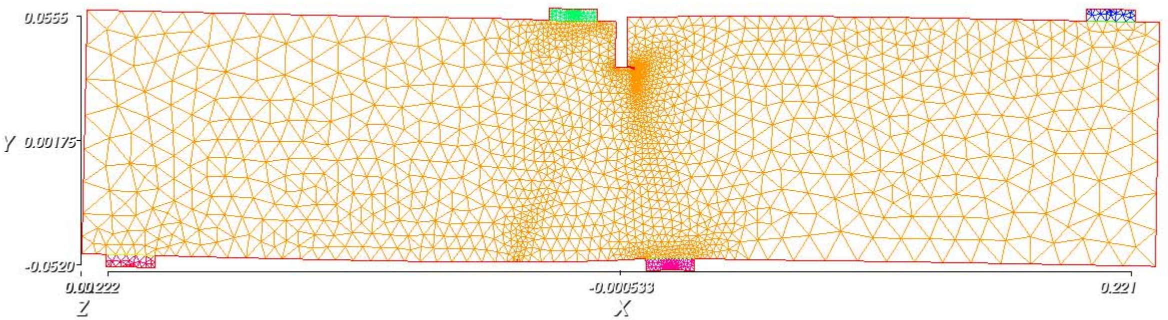




already cracked deformed specimen

Error-controlled crack propagation

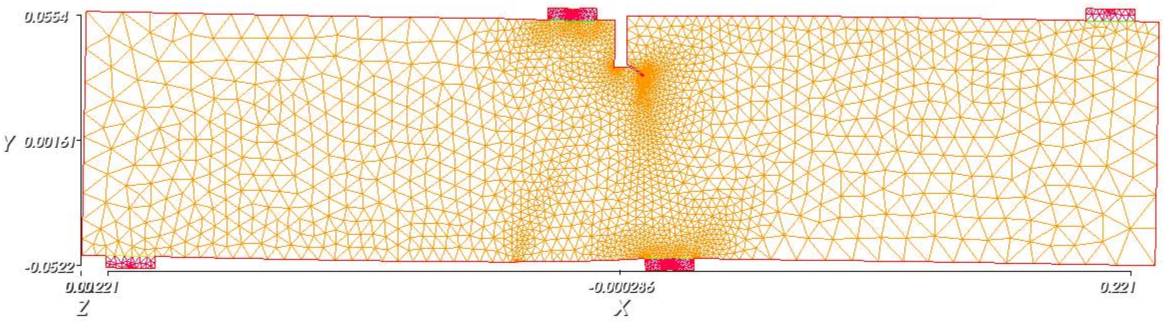
Loading Step 0 (peak load, already computed)



PropStep=0 (The LKM deformed, DOF=4350)

Error-controlled crack propagation

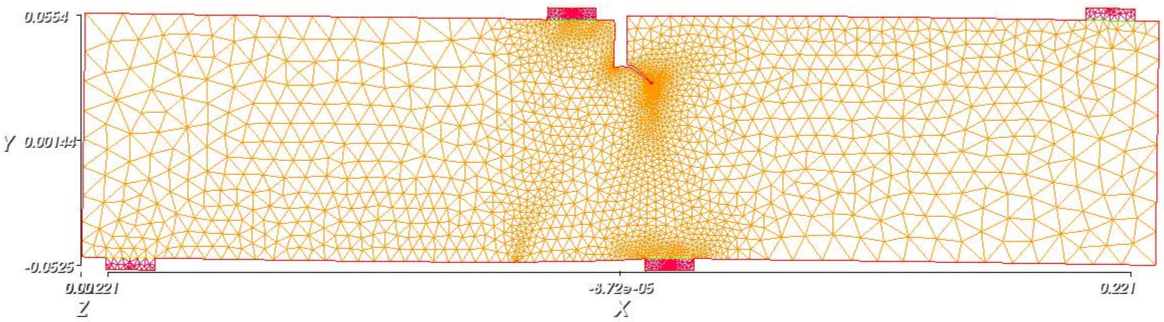
Loading Step 1



PropStep=10 (The LKM deformed, DOF=6156)

Error-controlled crack propagation

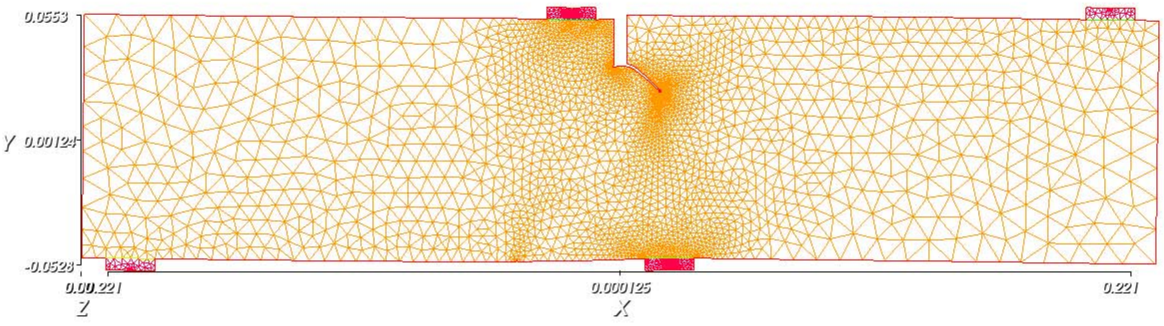
Loading Step 2



PropStep=20 (The LKM deformed, DOF=5942)

Error-controlled crack propagation

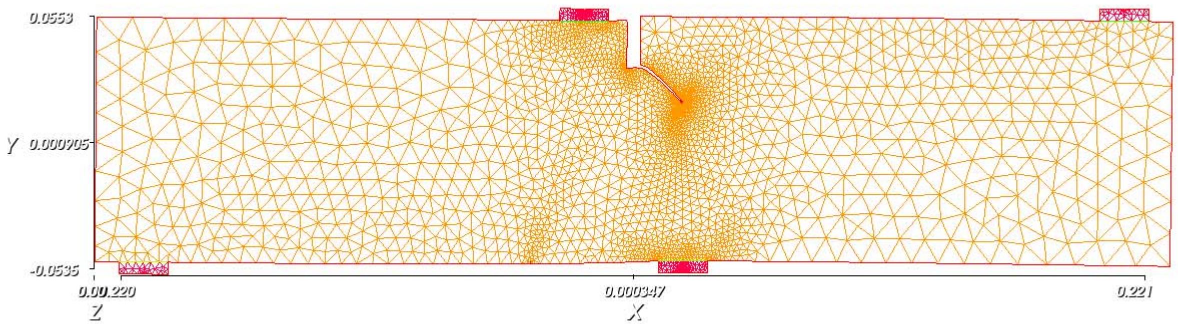
Loading Step 3



PropStep=30 (The LKM deformed, DOF=7588)

Error-controlled crack propagation

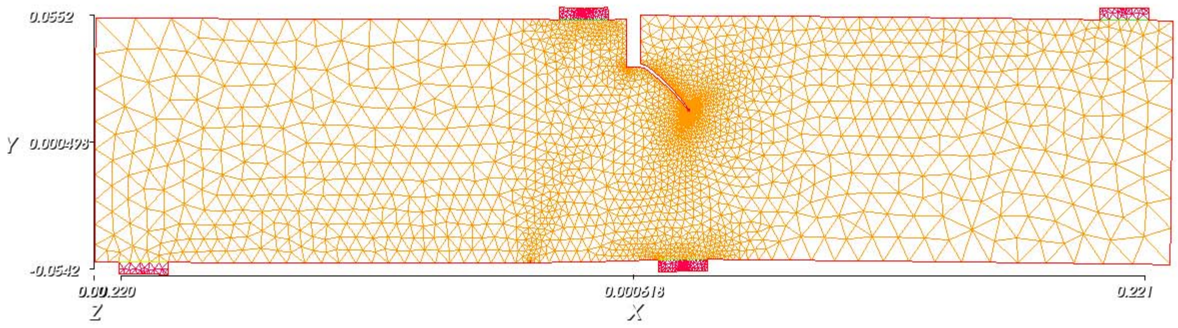
Loading Step 4



PropStep=40 (The LKM deformed, DOF=6114)

Error-controlled crack propagation

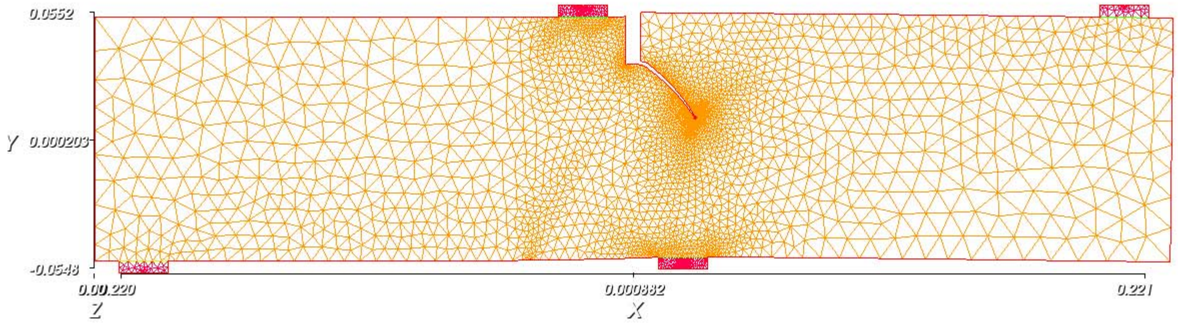
Loading Step 5



PropStep=50 (The LKM deformed, DOF=5860)

Error-controlled crack propagation

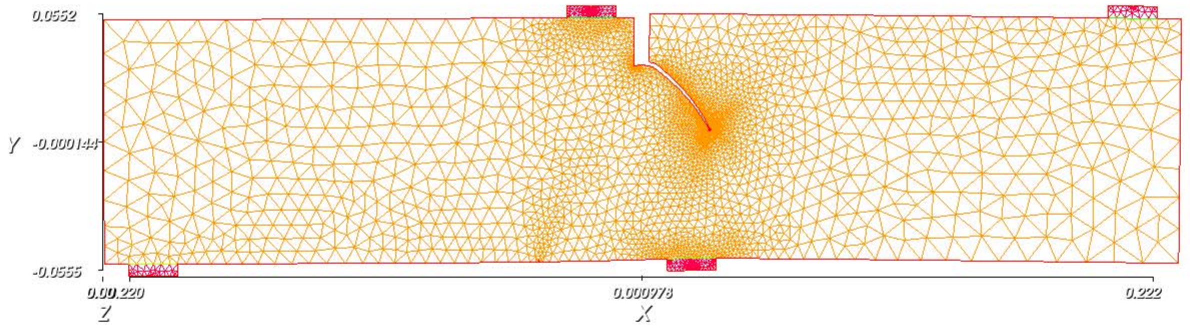
Loading Step 6



PropStep=60 (The LKM deformed, DOF=7212)

Error-controlled crack propagation

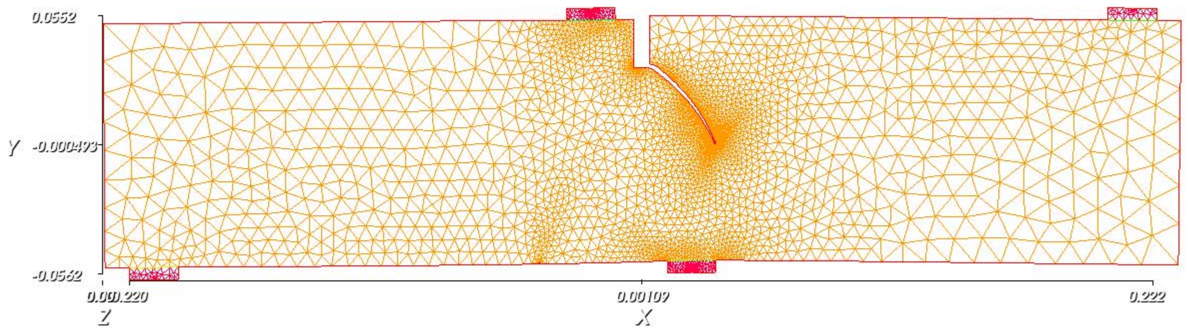
Loading Step 7



PropStep=70 (The LKM deformed, DOF=6870)

Error-controlled crack propagation

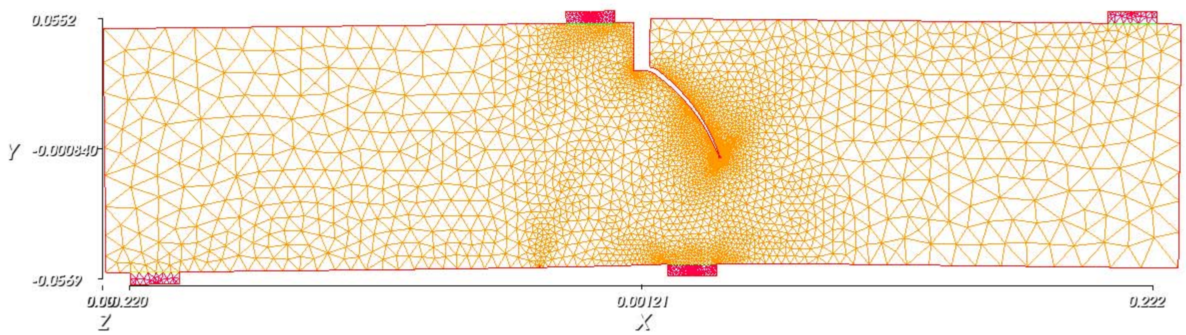
Loading Step 8



PropStep=80 (The LKM deformed, DOF=7320)

Error-controlled crack propagation

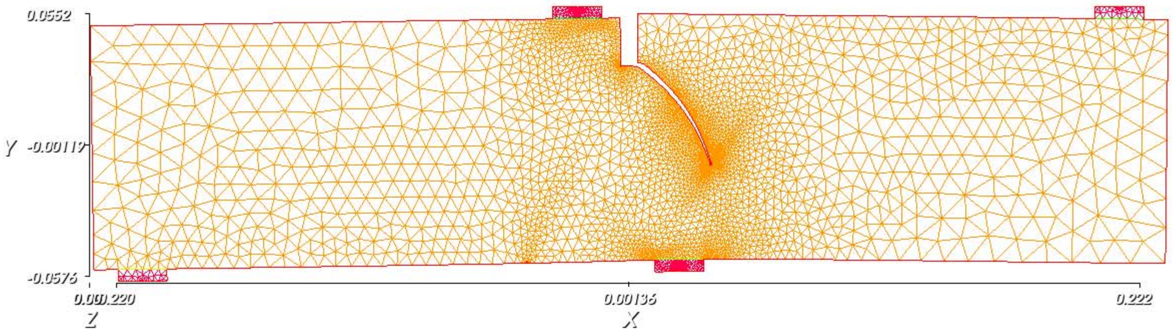
Loading Step 9



PropStep=90 (The LKM deformed, DOF=8102)

Error-controlled crack propagation

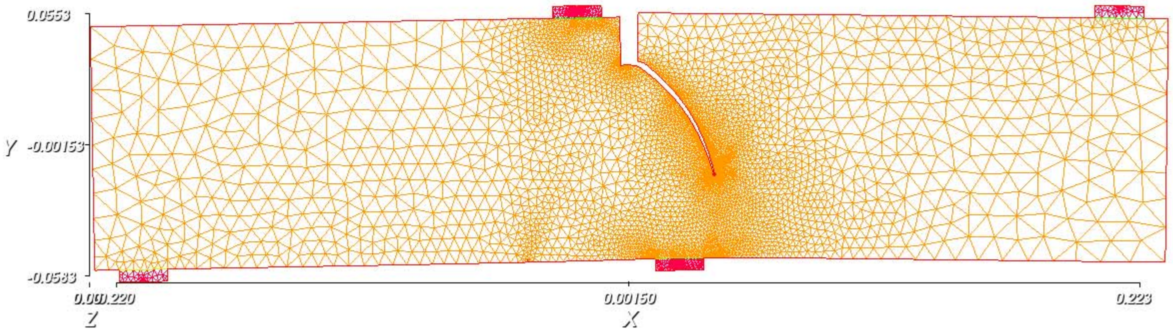
Loading Step 10



PropStep=100 (The LKM deformed, DOF=8464)

Error-controlled crack propagation

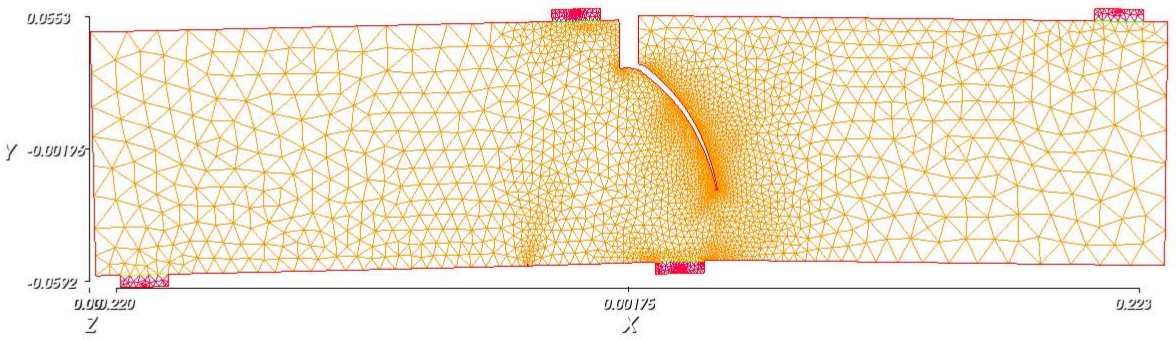
Loading Step 11



PropStep=110 (The LKM deformed, DOF=11112)

Error-controlled crack propagation

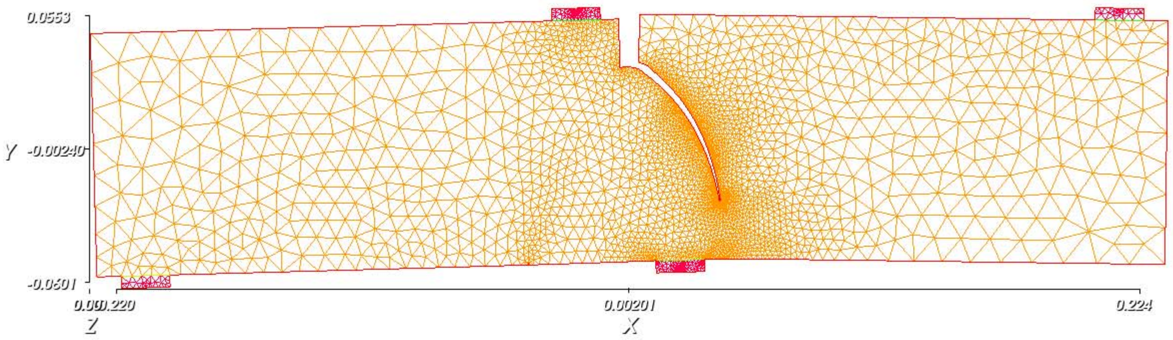
Loading Step 12



PropStep=120 (The LKM deformed, DOF=7108)

Error-controlled crack propagation

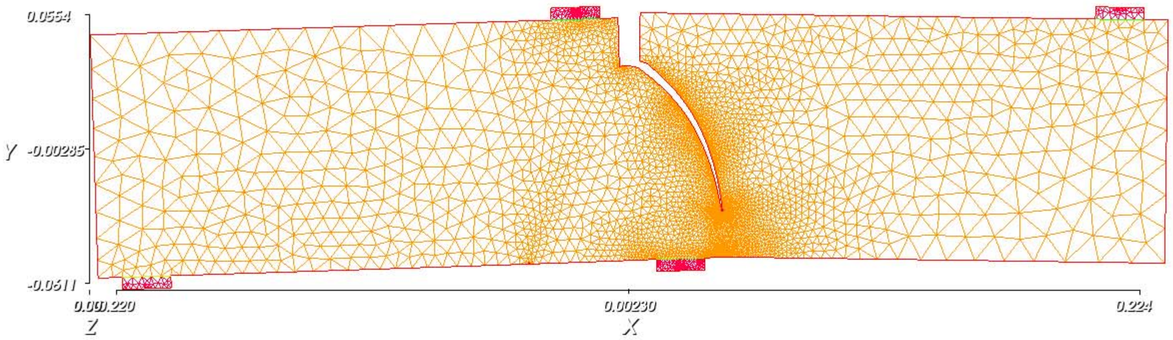
Loading Step 13



PropStep=130 (The LKM deformed, DOF=7086)

Error-controlled crack propagation

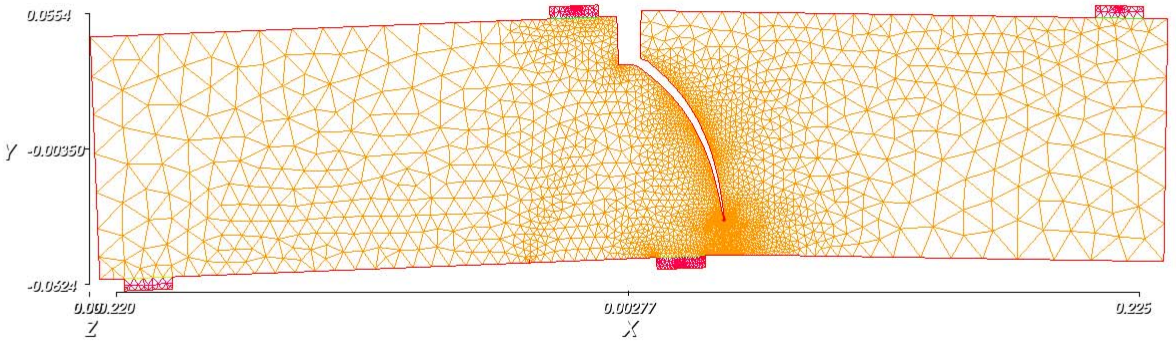
Loading Step 14



PropStep=140 (The LKM deformed, DOF=9138)

Error-controlled crack propagation

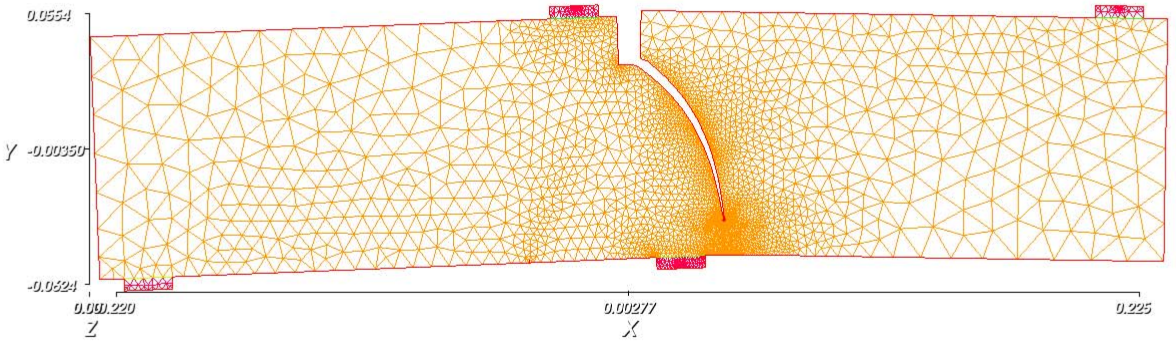
Loading Step 15 (we **stop** here)



PropStep=150 (The LKM deformed, DOF=8106)

Error-controlled crack propagation

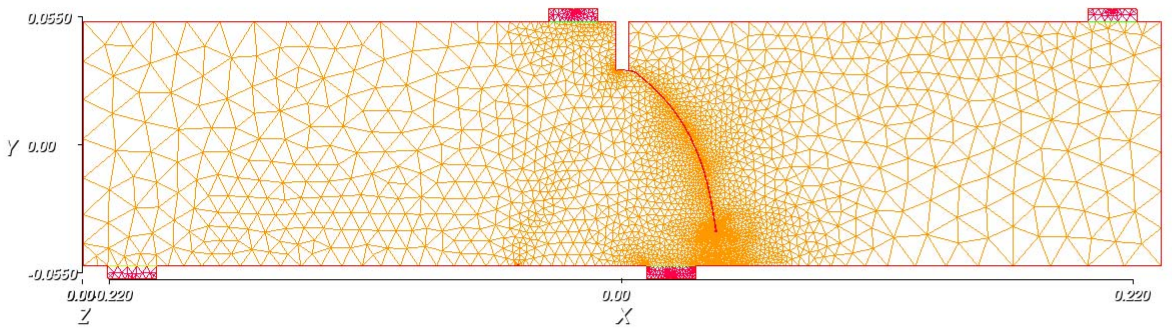
Loading Step 15 (we **stop** here)

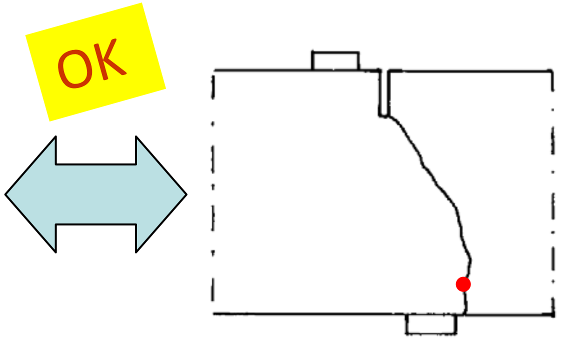


PropStep=150 (The LKM deformed, DOF=8106)

Computed crack pattern:

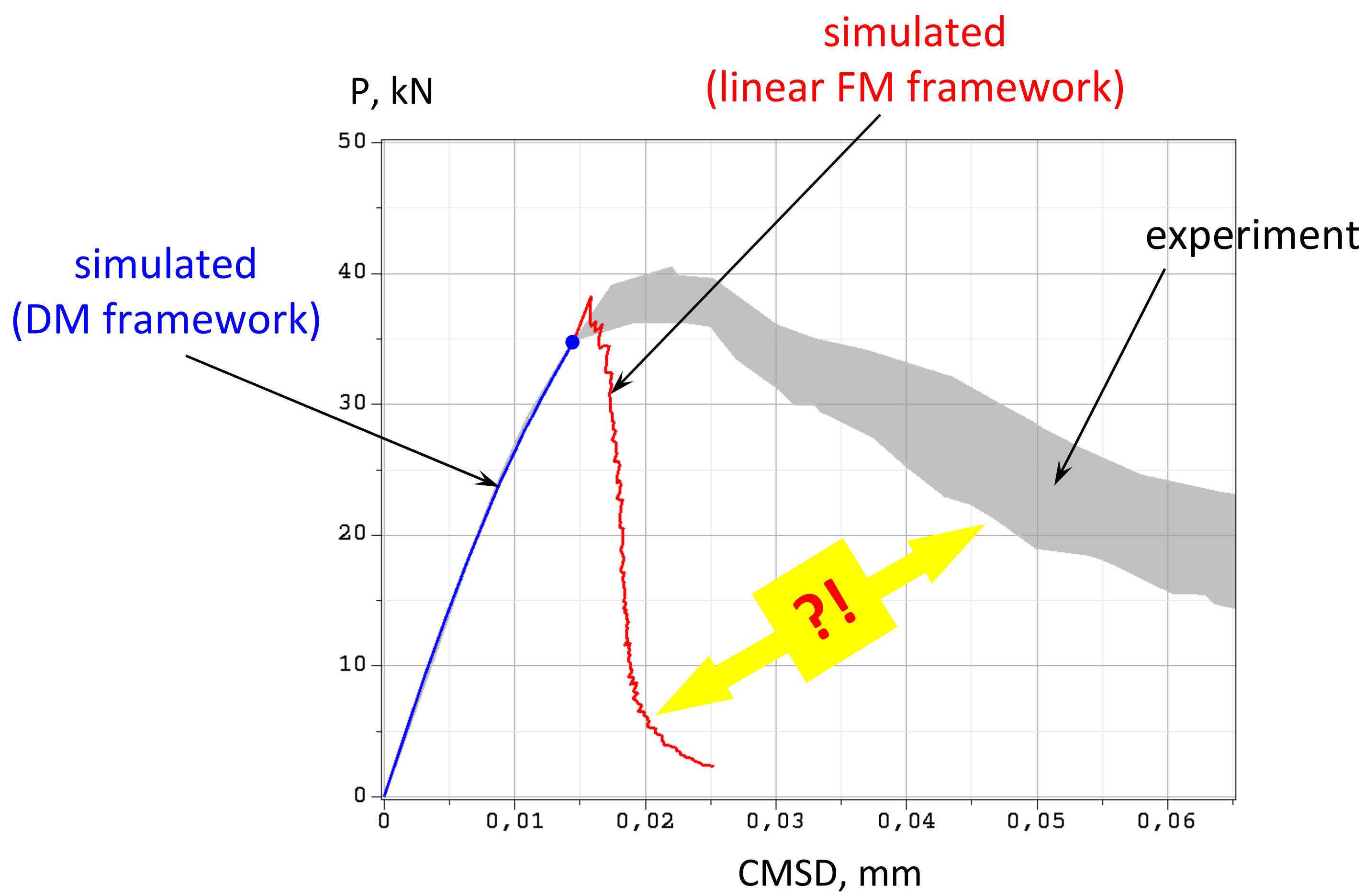
Schlangen (1993):





PropStep=150 (the last kept mesh in the code memory (LKM), DOF=8106)

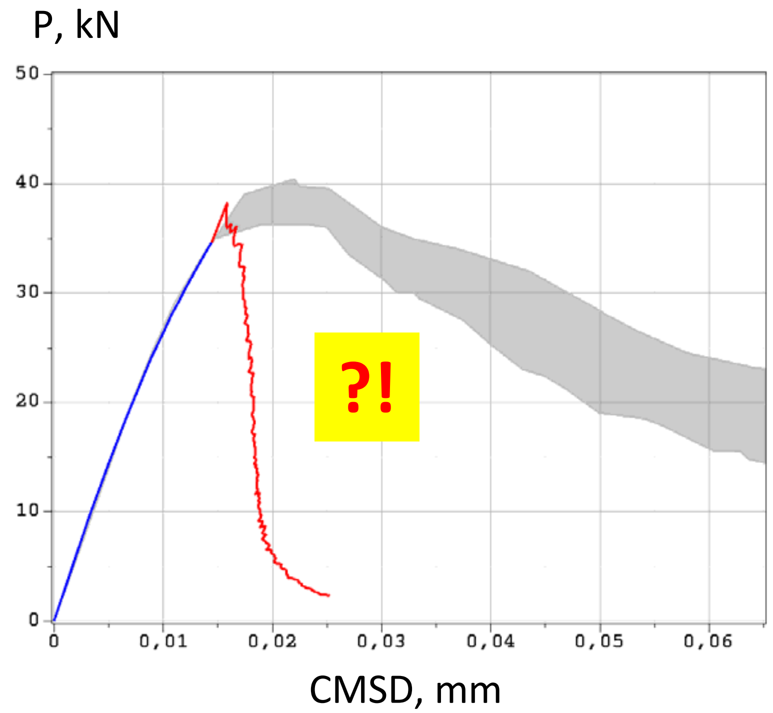
Error-controlled crack propagation

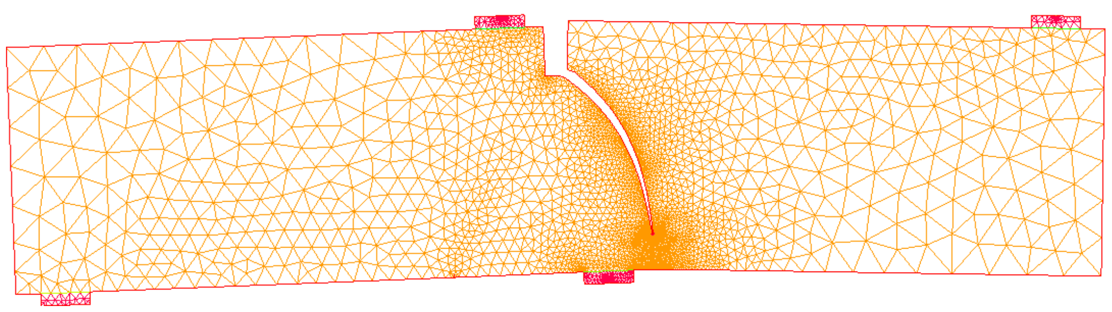


Error-controlled crack propagation

Secondary reasons for ?! :

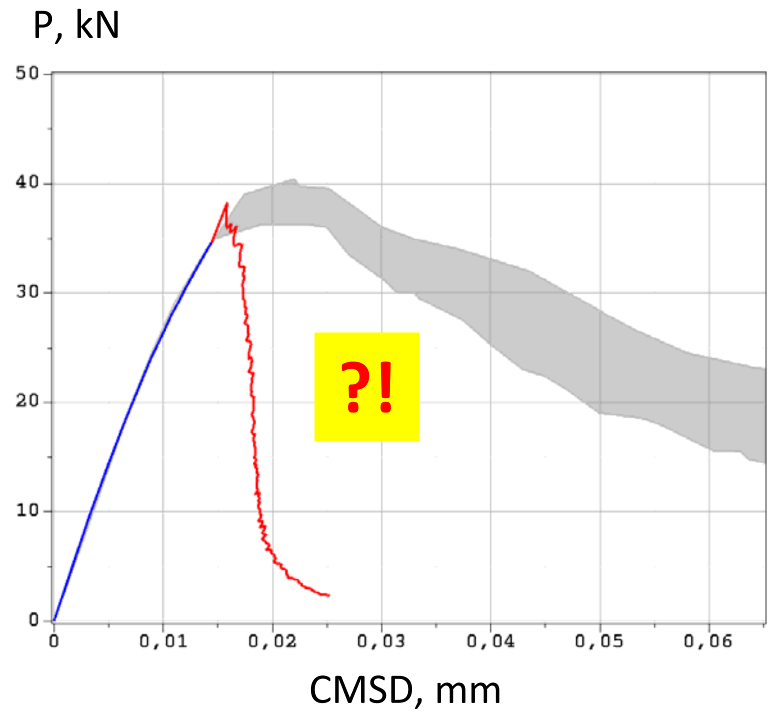
- friction between the platens and specimen is not accounted for
- J-integral computing may not be accurate enough

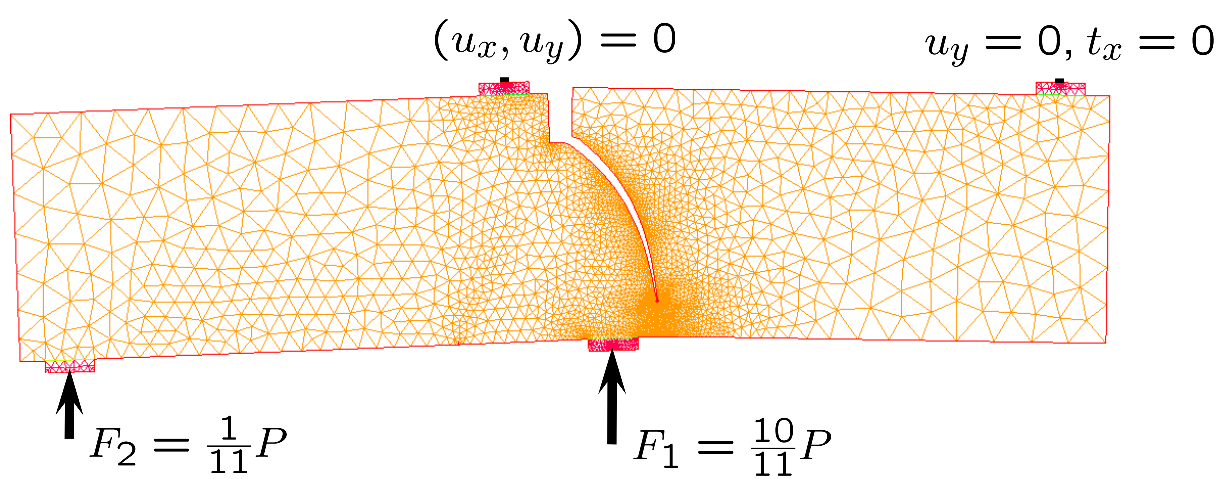




Error-controlled crack propagation

More importantly:



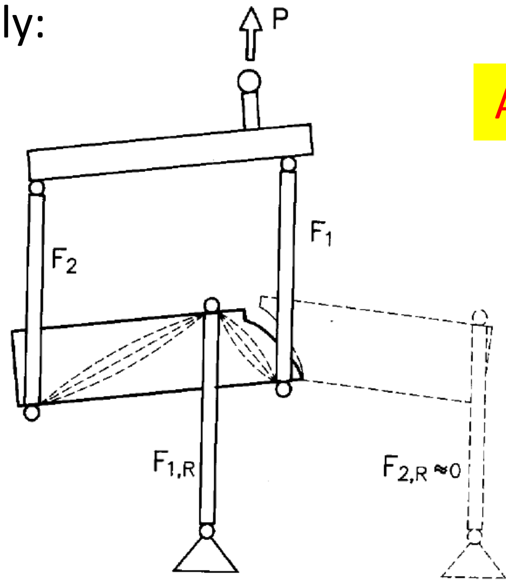


Assumed:

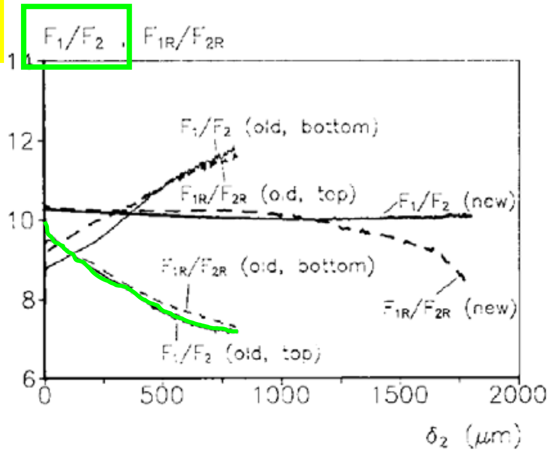
$$\frac{F_1}{F_2} = 10$$

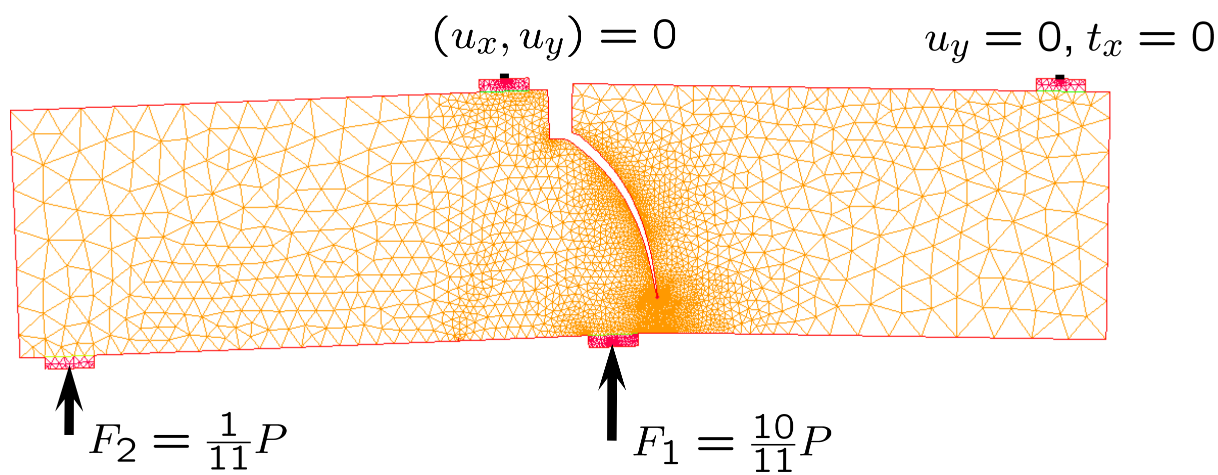
Error-controlled crack propagation

More importantly:



Actual:



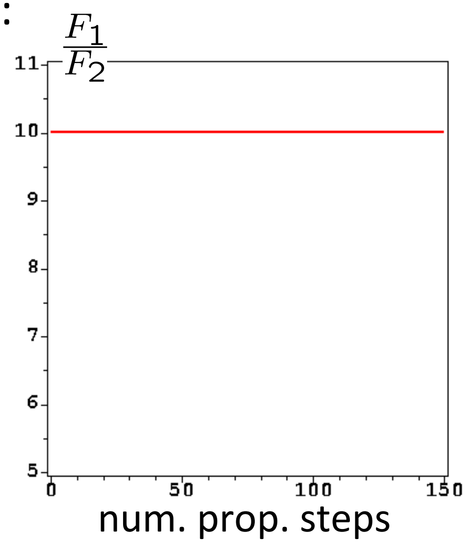


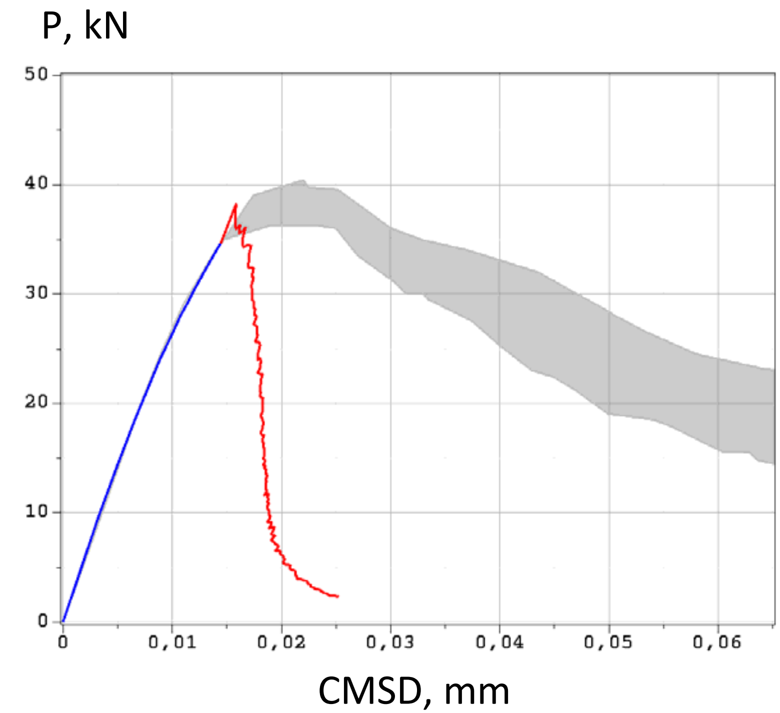
Assumed:

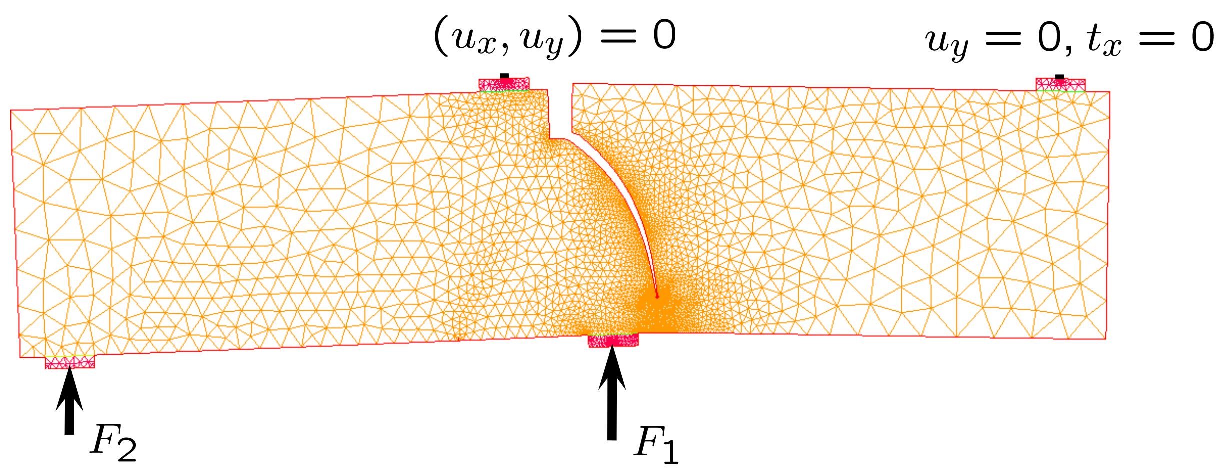
$$\frac{F_1}{F_2} = 10$$

Error-controlled crack propagation

More importantly:

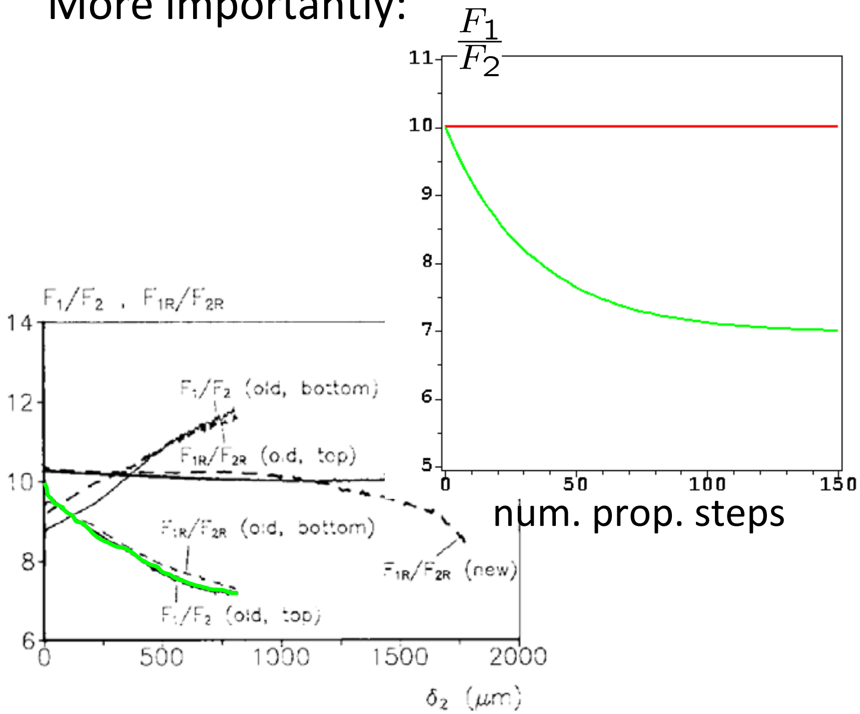


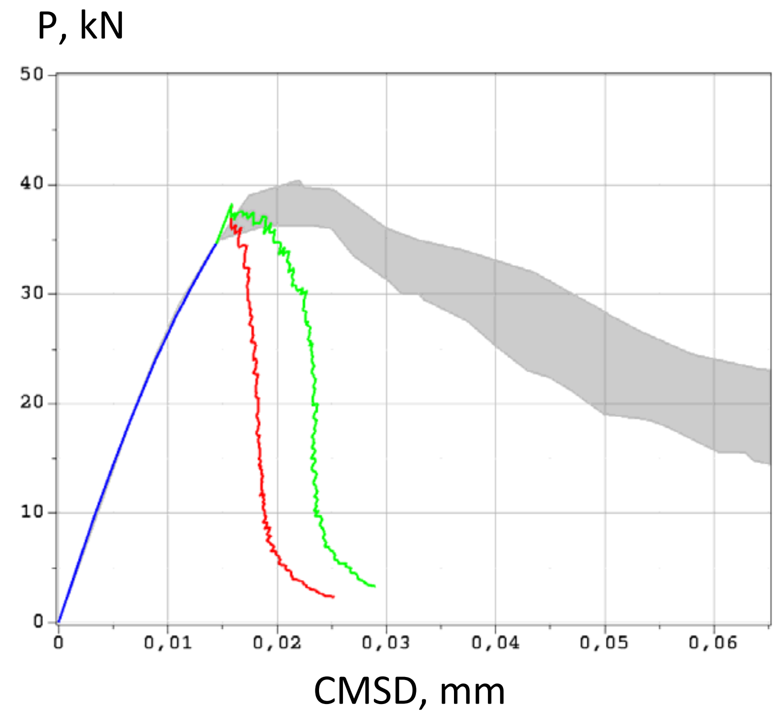


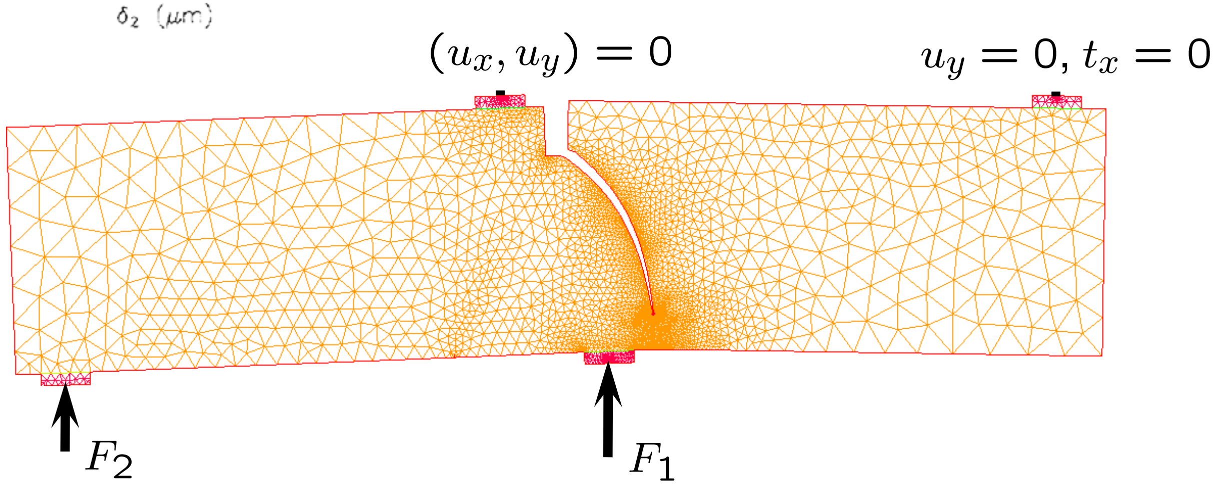


Error-controlled crack propagation

More importantly:

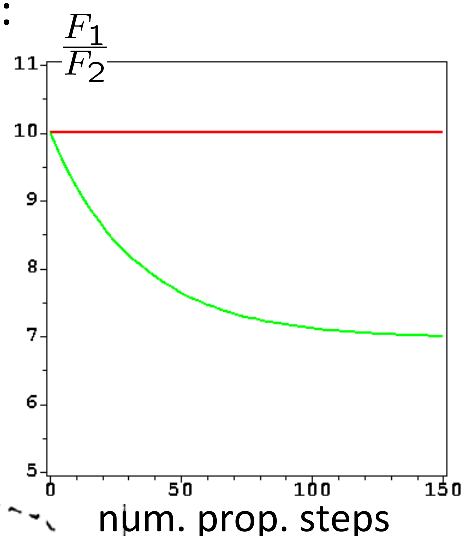


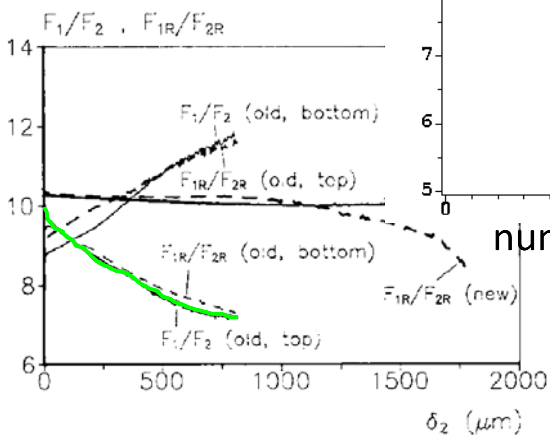


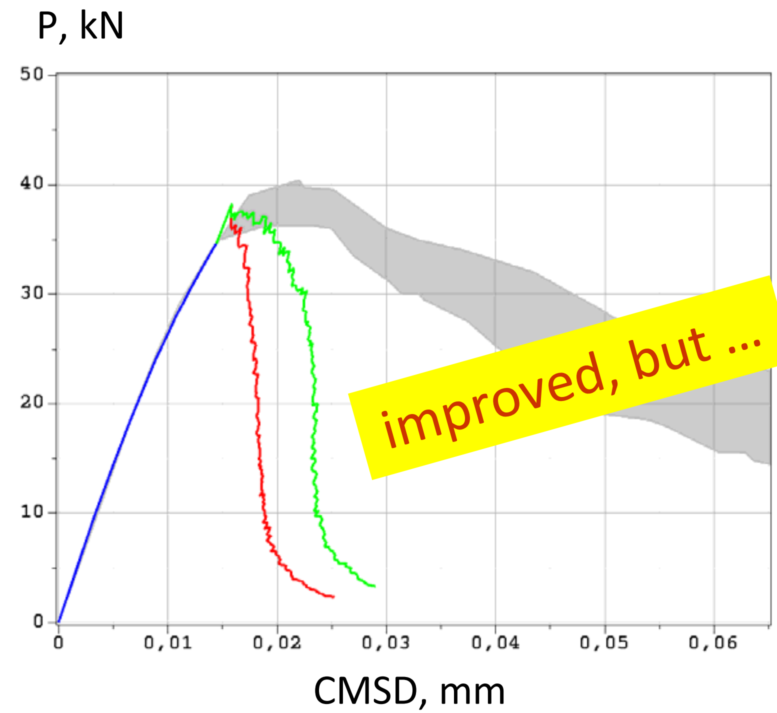


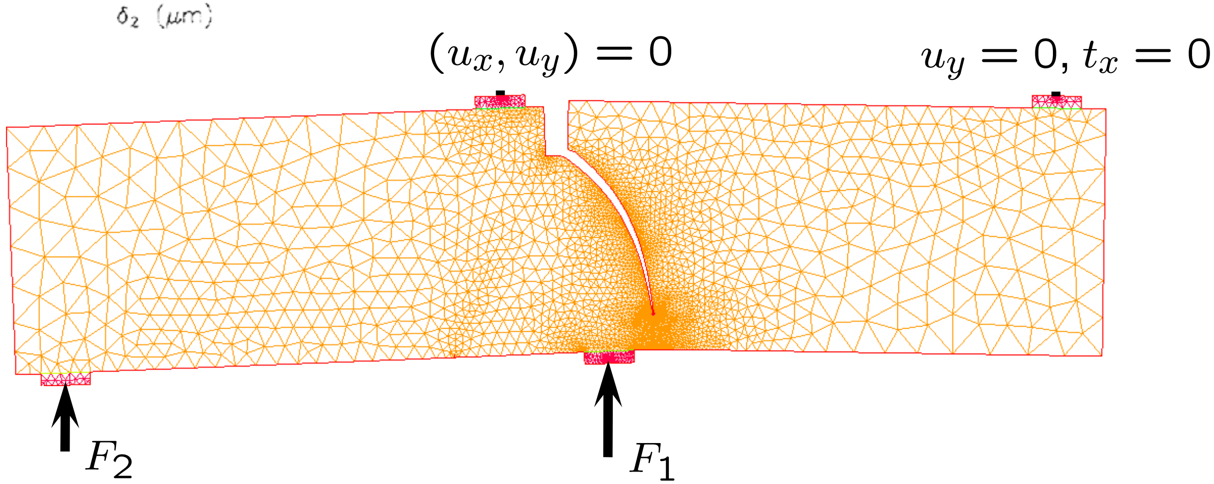
Error-controlled crack propagation

More importantly:



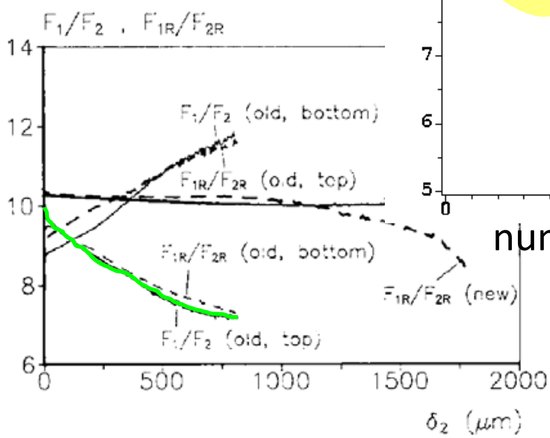


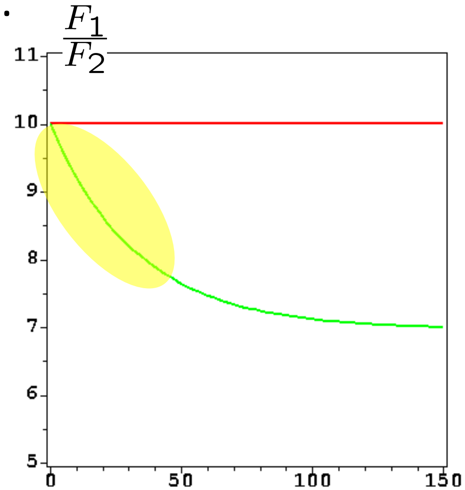


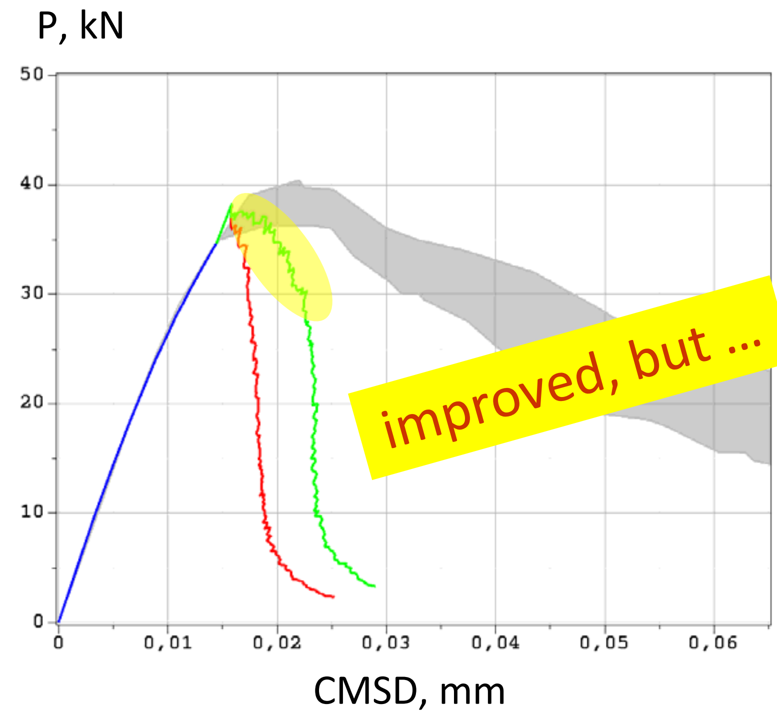


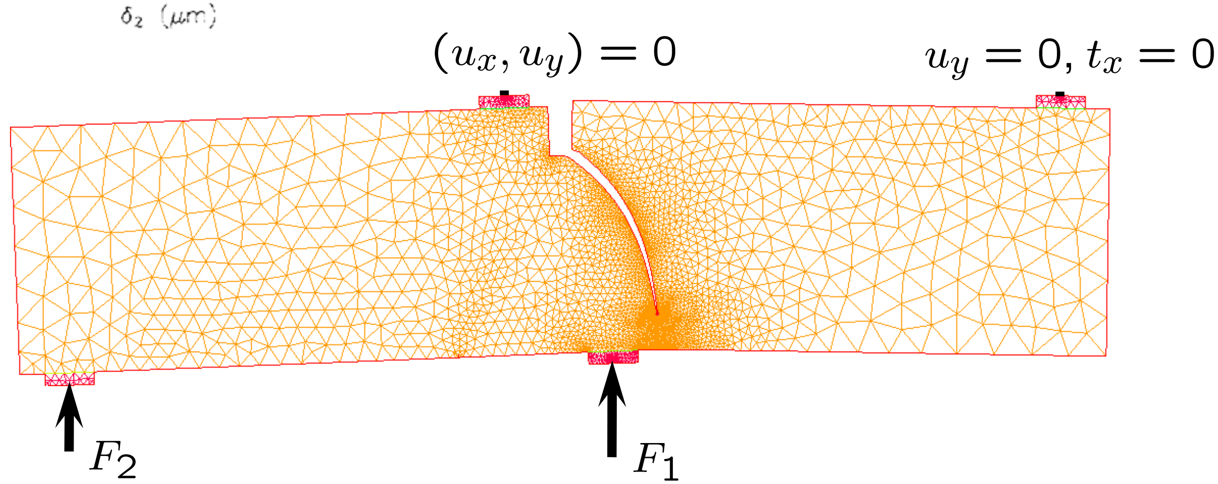
Error-controlled crack propagation

More importantly:



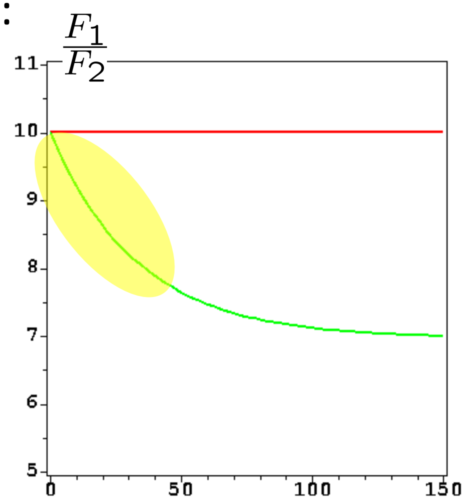




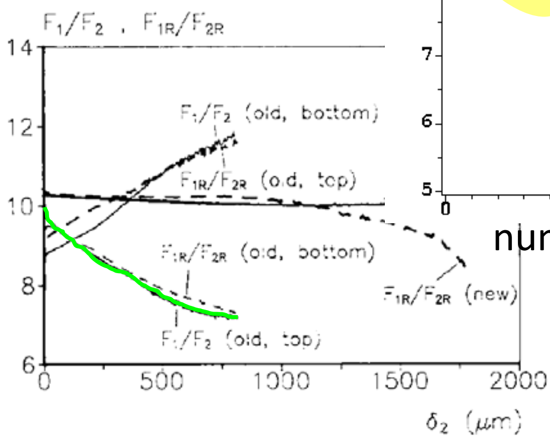


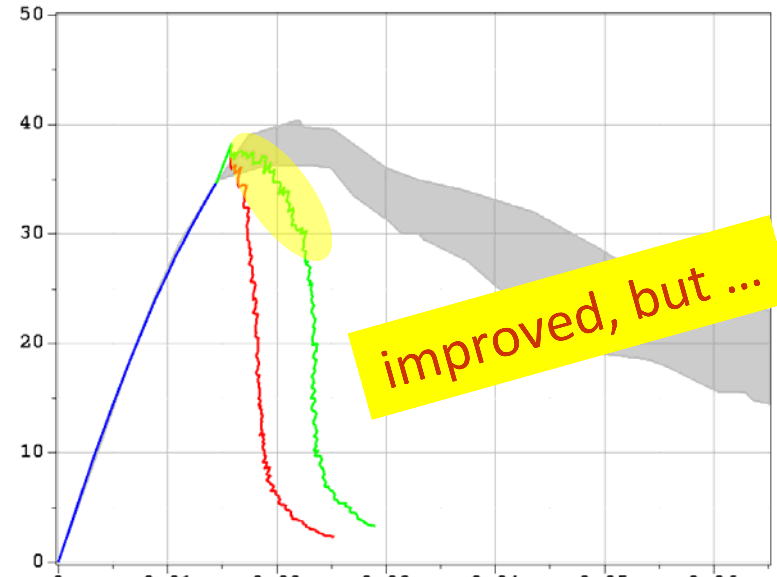
Error-controlled crack propagation

More importantly:

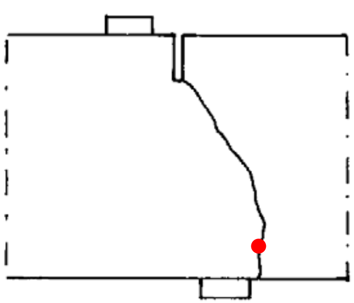


num. prop. steps



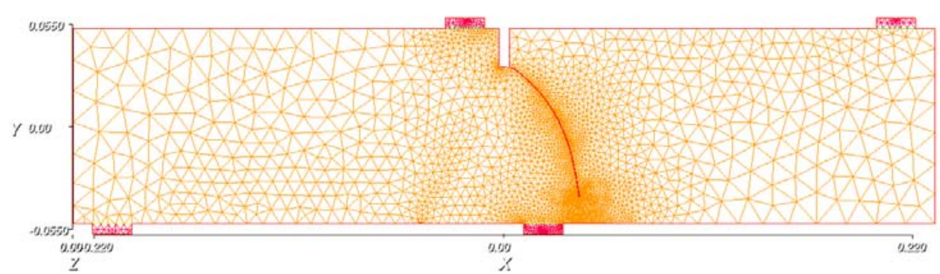


Schlangen (1993):



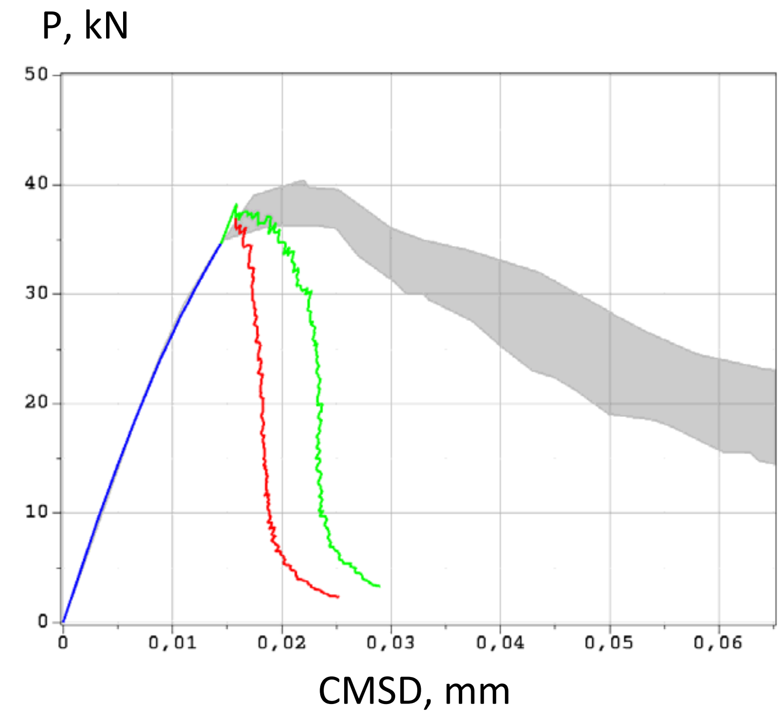
OK

Computed crack pattern:



PropStep=150 (the last kept mesh in the code memory (LKM), DOF=8106)

Conclusions:

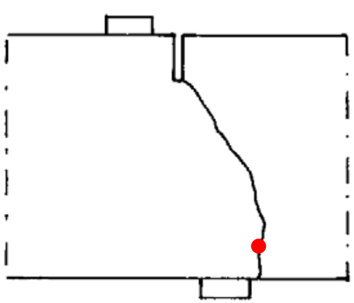


1. Correct qualitative prediction of
   - crack(s) nucleation, and
   - propagation

2. Simple, affordable and efficient manner

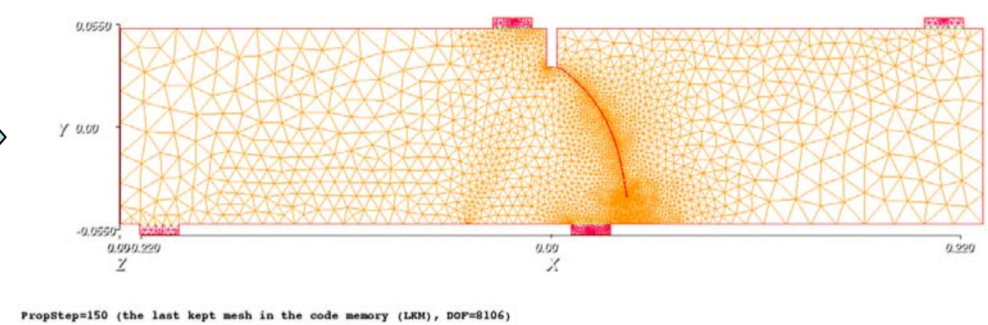
3. Failure in quantitative description in the post-peak regime (inappropriate choice of a framework)

Schlangen (1993):



OK

Computed crack pattern:



Conclusions:

1. Correct qualitative prediction of
   - crack(s) nu
2. Simple,
3. Failure in
   in the pos
   choice of a

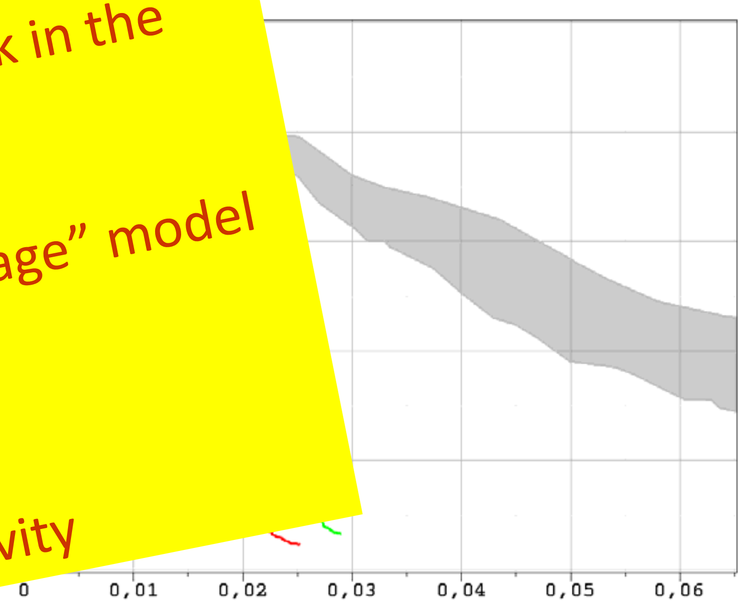
- Cohesive-zone model framework in the post-peak regime

  or

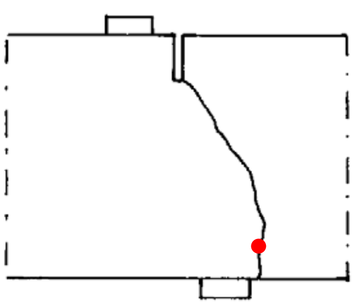
  "Phase-field" or "gradient damage" model for the entire failure process

  but, in any case,

- with error-controlled adaptivity

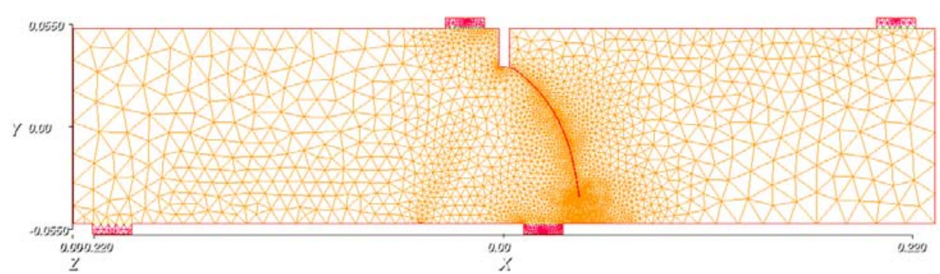


Schlangen (1993):



OK

Computed crack pattern:



PropStep=150 (the last kept mesh in the code memory (LKM), DOF=8106)